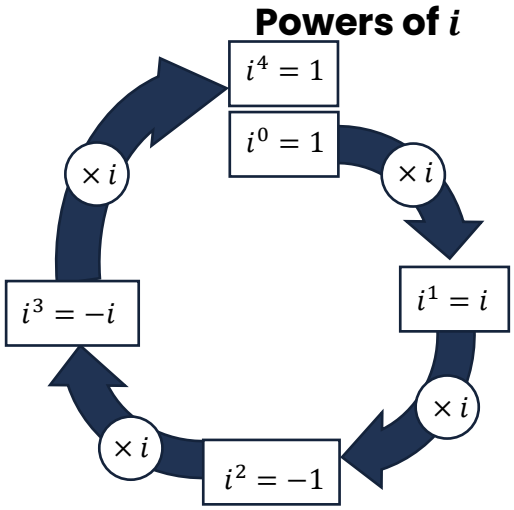
n^{th} roots and complex numbers

Pre-session Revision

Imaginary and complex numbers

For the activities in this workshop you will need to know

- That imaginary numbers are based on the square root of -1
- That the notation i is used for $\sqrt{-1}$ so $i = \sqrt{-1}$
- That any imaginary number can be written in terms of i e.g. $\sqrt{-4} = 2i$
- That complex numbers are numbers with a **real** and **imaginary** part (either of which could be 0)
- That complex numbers can be written in the form $z = x + yi$ where x is the real part of the number, $\mathrm{Re}(z)$ and y is the imaginary part of the number, $\mathrm{Im}(z)$



- $i^0 = 1, i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$ and so on, following the cycle $i, -1, -i, 1$
- $i^{-1} = -i, i^{-2} = -1\ , i^{-3} = i, i^{-4} = 1$ and so on

© MEI 2025

Addition and subtraction of complex numbers

To add two complex numbers, the real parts are added and the imaginary parts are added separately. The two results are combined to give the answer

e.g. $(3-2i)+(-1+5i)=(3+-1)+(-2+5)i=2+3i$

To subtract two complex numbers, the real parts are subtracted in order and the imaginary parts are subtracted in order separately. The two results are combined to give the answer

e.g. $(3-2i)-(-1+5i)=(3--1)+(-2-5)i=4-7i$

Calculations with Imaginary and Complex Numbers

The conjugate of a complex number

The conjugate of $z=x+yi$ is $z^*=x-yi$. Only the imaginary part changes its sign.

Multiplication of complex numbers

To multiply two complex numbers, the following procedure is followed

for $(a+bi)(c+di)$

multiply a by c to get ac

multiply $+bi$ by $+di$ to get $+bdi^2$ which is equal to $-bd$ (as $i^2=-1$)

multiply a by $+di$ to get $+adi$

multiply $+bi$ by c to get $+bci$

combine all of these to get $ac-bd+(ad+bc)i$

An example of multiplication

$$(3+2i)(2-5i)=6-10i^2-15i+4i$$
$$=6+10-11i$$
$$=16-11i$$

Division of complex numbers

To divide two complex numbers, the following procedure is followed

© MEI 2025

- write the division in the form $\frac{a+bi}{c+di}$
- multiply both top and bottom of the fraction by $c - di$ i.e. the conjugate of the denominator - the new denominator will be a real number
- separate the result into a real and an imaginary part (this isn't strictly necessary but with complex numbers it is better to be able to see the real and imaginary parts separately).

An example of division

$$(3 + 2i) \div (2 - 5i) = \frac{3 + 2i}{2 - 5i}$$

$$= \frac{3 + 2i}{2 - 5i} \times \frac{2 + 5i}{2 + 5i}$$

$$= \frac{6 + 10i^2 + 15i + 4i}{4 - 25i^2 + 10i - 10i}$$

$$= \frac{6 - 10 + 19i}{4 + 25}$$

$$= \frac{-4 + 19i}{29}$$

$$= -\frac{4}{29} + \frac{19}{29}i$$

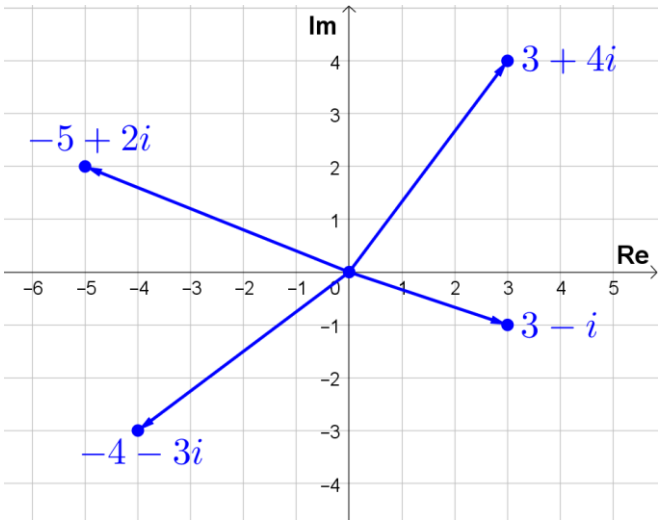
A geometrical interpretation of complex numbers

A complex number $z = x + yi$ can be represented geometrically on an Argand diagram.

This is a Cartesian grid with the horizontal axis representing the real part of the number and the vertical axis representing the imaginary part of the number.

The point (x, y) on an Argand diagram represents the complex number $x + yi$.

Complex numbers can also be represented by vectors on an Argand diagram.



© MEI 2025