

Matrix Multiplication

Pre-session Revision

Matrix Multiplication

For this workshop you will need to be confident that you can multiply two matrices together.

Here is a reminder.

1. The order of a matrix

$\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$ is a 2×2 matrix.

It has 2 rows: $\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$

and 2 columns: $\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$

The order of the matrix is 2×2 . Rows are always written first and columns second.

$\begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -1 \end{pmatrix}$ is a 2×3 matrix.

This matrix has 2 rows and 3 columns.

The order of the matrix is 2×3

For matrix multiplication, order is important

To be able to multiply two matrices together, the number of **columns** in the first matrix **must** be the same as the number of **rows** in the second. If this is not the case then matrix multiplication can not be done.

If two matrices are of orders that allow for multiplication then they are said to be **conformable** under matrix multiplication.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$$

$2 \times 3 \quad 3 \times 1$

The first matrix is order 2×3 , the second is order 3×1 . The two matrices may be multiplied.

To perform a quick test of conformability, you can write the orders of each matrix down, first matrix before second like this:

$$2 \times 3 \quad 3 \times 1$$

Summary

Conformable...can be multiplied	Non-conformable...cannot be multiplied
$\begin{pmatrix} -2 & 3 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ $2 \times 2 \quad 2 \times 2$	$\begin{pmatrix} -2 & 3 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & -1 \\ 0 & 3 \end{pmatrix}$ $2 \times 2 \quad 3 \times 2$

2. The mechanism for matrix multiplication

Matrix multiplication involves multiplying every element in a particular **row** of the **first** matrix by a corresponding element in a particular **column** of the **second** matrix. The results of these multiplications are summed.

The row and column used dictate the position of the element found by doing this in the resulting matrix. As long as you can identify the position of an element in the answer matrix, you know which row and column are multiplied and summed to find it.

Most of the matrices you are going to multiply together are 2×2 or $2 \times n$ matrices.

This diagram shows you how the rows and columns of the two matrices being multiplied relate to the positions of the elements in the answer:

$$\begin{array}{l} \text{row 1} \rightarrow \\ \text{row 2} \rightarrow \end{array} \begin{pmatrix} 1 & -2 \\ -1 & 2 \end{pmatrix} \begin{array}{l} \text{col 1} \\ \text{col 2} \end{array} \begin{pmatrix} -2 & 3 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix}$$

$a_{1,1}$ is the result of multiplying row 1 (of the first matrix) by column 1 (of the second).

$$a_{1,1} = (1 \times -2) + (-2 \times 2) = -6$$

Find the values of $a_{1,2}$, $a_{2,1}$ and $a_{2,2}$. The answers are on the next page.

$$a_{1,2} = (1 \times 3) + (-2 \times 0) = 3$$

$$a_{2,1} = (-1 \times -2) + (2 \times 2) = 6$$

$$a_{2,2} = (-1 \times 3) + (2 \times 0) = -3$$

$$\begin{pmatrix} 1 & -2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} -6 & 3 \\ 6 & -3 \end{pmatrix}$$

3. The order of the answer matrix

You can use the orders of the matrices being multiplied together to find the order of the resulting matrix.

$$\begin{pmatrix} 2 & 3 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 4 & -1 \\ -3 & 2 & 1 & -2 \end{pmatrix}$$

$$2 \times \text{2 conformable 2} \times 4$$



The first matrix has 2 rows and the second matrix has 4 columns.

The order of the resulting matrix will be 2×4 .