COURSE SAMPLE

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SUSTAINABLE ELECTRICAL POWER SYSTEMS ENGINEERING

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AC CIRCUIT ANALYSIS AND POWER FOR POWER SYSTEMS

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CONTENTS

PREREQUISITES

1. DC CIRCUIT ANALYSIS

- Describe the relationship between basic electrical terms: power, energy, voltage, current
- Be able to simplify networks of resistors
- Be able to calculate the DC current/voltage/power in resistive circuits (being able to use KVL/KCL/superposition is helpful but not strictly required).
- Describe the structure and function of a resistor, inductor, and capacitor
- Describe how the instantaneous charge, voltage, current and power in/through these components (R, L, C) changes when connected to a DC source.

2. MATHS

- Trigonometry
- Vectors
- Complex numbers, including exponential and polar form
- To follow the derivations, you will need to differentiate using the chain rule.

STEADY STATE CIRCUIT ANALYSIS

Electric power systems operate primarily in sinusoidal steady state. This means that all currents and voltages inside a power network are time varying sinusoids with a constant magnitude, phase angle, and frequency[1]. Thus, the general equation for voltage and current at any point in a power system is defined as:

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

Symbol	Name	Units
θ_v or θ_i	Phase angle	Radians (or degrees)
V_m or I_m	Peak voltage/current magnitude	Volts or Amps
$\omega = 2\pi f$	Angular frequency	Radians per second
t	Time	Seconds
$f = \frac{1}{T}$	Frequency (generally 50 or 60 Hz)	Hertz (1/seconds)
T	Period	Seconds

1. PHASORS

Assuming that the power system is (approximately) operating in steady state we can simplify the analysis of the system by using phasors – this allows use to use much simpler algebraic equations for the calculation of voltage, current and power instead of differential equations. When using phasors we will use the RMS values for voltage and current

$$(V = V_{rms} = \frac{V_m}{\sqrt{2}}, I = I_{rms} = \frac{I_m}{\sqrt{2}}),$$

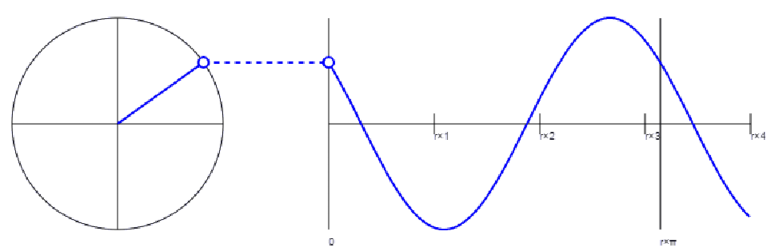
we will also use bold symbols to denote complex numbers ($\boldsymbol{V}$).

Thus the phasors corresponding to the equations above are given by:

$$\boldsymbol{V} = Ve^{j\theta_v} = V\angle\theta_v$$

$$\boldsymbol{I} = Ie^{j\theta_i} = I\angle\theta_i$$

What is a phasor?

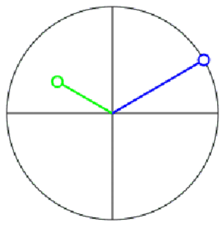


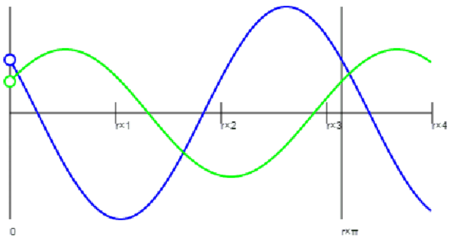
A sinewave is defined as the projection of a point moving at a uniform speed around a circle, onto an axis. (imagine the point rotating around the circle)

If you had multiple sinewaves at the same frequency, the points moving around the circle would be at a fixed angle away from each other and each have a constant distance from the origin.

[1] Later you will work out how to analyze systems with faults and harmonics, both of which break this simplifying assumption.

If we are in steady state then all the information we care about is represented by where the points on the circle are relative to each other - we care about the magnitude of each voltage and the angle of each voltage relative to some reference voltage.





A phasor is a complex number that encodes a magnitude and angle, it throws away the frequency (f) and the time (t), as these are not relevant to our analysis once everything is in this form.

The mathematical validity for this comes from Euler's formula:

$$\cos(\omega t + \theta) = \frac{1}{2}\left[e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right]$$

Showing that we can transform between a cosine and a complex exponential and back.

Why use phasors?

The relationship between voltage and current in a circuit containing just a resistor is simply:

$$v(t) = R \times i(t)$$

For an inductor the voltage is proportional to the change in current and for a capacitor the voltage is proportional to the integral of the current as given by:

$$v(t) = L\frac{di(t)}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$$

This means that to find the equation for instantaneous voltage, $v(t)$, in an AC circuit containing an inductor or capacitor you need to use differential equations. In a circuit with more than a few elements this quickly becomes unwieldy to solve by hand.

What we end up with if we do solve the full differential equations is a transient term that decays rapidly and a term that is a phase shifted and scaled value. When we look at the impedance we will just see this second term, the transient is ignored by our simplification, but this is part of what steady-state means, we assume transients have decayed to zero.

RMS – root-mean-square values

(see https://en.wikipedia.org/wiki/Root_mean_square)

In the definition of the instantaneous voltage and current above we used the peak magnitudes for voltage and current, in power systems we normally work with the RMS value. The RMS value is defined as:

$$A_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} [u(\tau)]^2\, d\tau}$$

For sinusoids this simplifies to:

$$A_{rms} = \frac{A}{\sqrt{2}}$$

The reason we use RMS is, for AC systems, RMS is equal to the value of the direct current that would produce the same average power dissipation in a resistive load. This allows us to define the power of an AC system as $V_{rms} \times I_{rms}$, rather than the more clumsy $\frac{V_m I_m}{2}$.

Reference voltage angle

In the description of a phasor I said we only care about the voltage relative to a reference. We can choose one voltage in the system to be at an angle of zero and just define everything else in reference to that voltage. It makes no difference to the results but can make the analysis easier to carry out.

Calculations using phasors

$j^2 = -1 \qquad {}^{1}/_{j} = -j$

$(M_1\angle\theta_1) \times (M_2\angle\theta_2) = (M_1M_2)\angle(\theta_1 + \theta_2)$

$(a_1 + jb_1) + (a_2 + jb_2) = (a_1 + a_2) + j(b_1 + b_2)$

If $\boldsymbol{Z} = a + jb = |Z|e^{j\varphi} = |Z|\angle\varphi$ then

$Re\{Z\} = a = |Z|\cos\varphi$

$Im\{Z\} = b = |Z|\sin\varphi$

$|Z| = \sqrt{a^2 + b^2}$ (Magnitude)

$\varphi = \tan^{-1}\left({}^{b}/_{a}\right)$ (Angle)

$\boldsymbol{Z}^* = a - jb = |Z|e^{-j\varphi}$(conjugate)

Phasors questions

1. Convert the following into a single phasor of the form $x\angle\theta$:
   a. $10 + j3 = 10.44\angle 16.70°$
   b. $4e^{j\pi/6}$ (the angle is in radians)
   c. $(1 + 0.7j) \times (0.3 + 1.2j)$
   d. $(1 + 0.7j) + (0.3 + 1.2j)$
   e. $(10 + j3)/(1 - j)$
   f. $100\cos(\omega t + 30°) = 70.71\angle 30°$
   g. $100\sin(\omega t) = 70.71\angle -90°$
   h. $100\sin(\omega t + 30°)$
   i. $100\cos(100\pi t - 30°)$
   j. $80\sin(\omega t + 30°) + 20\cos(\omega t + 30°)$
   k. $(0.96\angle 18.19°) \times (1.12\angle -3.19°)$
2. Using phasors express $v(t)$ as a single cosine function:

$$v(t) = 20\cos(\omega t) + 10\cos\left(\omega t + \frac{\pi}{3}\right) - 5\cos\left(\omega t - \frac{\pi}{9}\right)$$

2. IMPEDANCE

In the same way that we did with the simple circuits we want to know the relationship between voltage and current in resistors, inductors and capacitors. This value is called impedance, as it is the ratio of voltage to current it is defined as:

$$\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{V}{I}\angle(\theta_v - \theta_i)$$

This is similar to the concept of resistance in DC circuits. It has to represent phase shifts as well as a scaling in magnitude; thus the impedance has a magnitude and angle (or alternatively a real and imaginary part):

$$\boldsymbol{Z} = R + jX = |Z|e^{j\varphi}$$

Where the angle between voltage and current (phi) is defined as:

$$\varphi = (\theta_v - \theta_i)$$

It's not obvious that there should be such a simple relationship in an arbitrarily sized network of components defined by differential equations, but if you follow the derivation you will see that it holds true[2].

There are names given to each element of the impedance and its inverse (called the admittance). These are as follows:

Symbol	Name	Units
$\boldsymbol{Z} = R + jX$	Impedance	Ω
$Re\{Z\} = R$	Resistance	Ω
$Im\{Z\} = X$	Reactance	Ω
$\boldsymbol{Y} = {}^{1}/_{\boldsymbol{Z}} = G + jB$	Admittance	S
$Re\{Y\} = G$	Conductance	S
$Im\{Y\} = B$	Susceptance	S

The table below gives the relationship between the voltage and current in resistors, inductors and capacitors in any steady state AC system.

Time Domain	Complex Domain	Impedance
$v(t) = R \times i(t)$	$\boldsymbol{V} = R\boldsymbol{I}$	$Z_R = R$
$v(t) = L\frac{di(t)}{dt}$	$\boldsymbol{V} = j\omega L\boldsymbol{I}$	$\boldsymbol{Z_L} = j\omega L$
$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$	$\boldsymbol{V} = \boldsymbol{I}\frac{-j}{\omega C}$	$\boldsymbol{Z_C} = \frac{-j}{\omega C} = \frac{1}{j\omega C}$

[2] Here we are assuming that all components are linear (e.g. ideal resistors, capacitors and inductors; ideal sources but not diodes or transistors [wiki]) and that we are operating in steady state (e.g. a long time after the components have been connected, without flipping switches, or changing the current/voltage sources).

Derivation of impedances

Resistors
The equation for the voltage/current relationship of a resistor is simply:

$$v(t) = R \times i(t)$$
$$\frac{v(t)}{i(t)} = R$$
$$\frac{\boldsymbol{V}}{\boldsymbol{I}} = R$$

Capacitor
The equation for the voltage/current relationship of a capacitor is given below (see https://en.wikipedia.org/wiki/Capacitor):

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau \qquad \therefore \qquad i(t) = C\frac{dv(t)}{dt}$$

Substitute the instantaneous voltage; solve using the chain rule with $u = \omega t + \theta_v$, $y = \cos u$:

$$i(t) = C\frac{d[V_m \cos(\omega t + \theta_v)]}{dt} = CV_m \times [(\omega) \times (-\sin(\omega t + \theta_v))] = -\omega C V_m \sin(\omega t + \theta_v)$$

Convert back to a cosine using the identity $\sin(\omega t + \theta + 90°) \equiv \cos(\omega t + \theta)$:

$$i(t) = -\omega C V_m \cos(\omega t + \theta_v - 90°)$$
$$\frac{v(t)}{i(t)} = \frac{V_m \cos(\omega t + \theta_v)}{-\omega C V_m \cos(\omega t + \theta_v - 90°)}$$

Convert to phasors (using exponential form) to simplify:

$$\boldsymbol{Z_C} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{Ve^{i\theta_v}}{-\omega CVe^{i\theta_v - 90°}} = \frac{V}{-\omega CV}e^{i(\theta_v - \theta_v + 90°)} = \frac{e^{i90°}}{-\omega C} = \frac{-j}{\omega C}$$

Inductor
The equation for the voltage/current relationship of an inductor is given below:

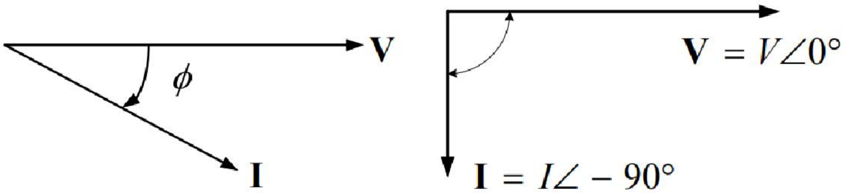
$$v(t) = L\frac{di(t)}{dt}$$

Substitute the instantaneous current; solve the same way as for a capacitor:

$$v(t) = L\frac{d[I_m \cos(\omega t + \theta_i)]}{dt} = LI_m[(\omega) \times (-\sin(\omega t + \theta_v))] = \omega L I_m \cos(\omega t + \theta_v + 90°)$$
$$\frac{v(t)}{i(t)} = \frac{\omega L I_m \cos(\omega t + \theta_v + 90°)}{I_m \cos(\omega t + \theta_i)}$$
$$\boldsymbol{Z_L} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{\omega L I e^{i\theta_v - 90°}}{Ie^{i\theta_v}} = \frac{\omega L I}{I}e^{i(\theta_v - \theta_v + 90°)} = \omega L e^{i90°} = j\omega L$$

3. PHASOR DIAGRAMS

Phasors can be plotted in the complex plane against each other to graphically depict how they relate, this is particularly useful to visualize the angle between voltage and current, φ. Two example phasor diagrams are shown below:



4. STEADY STATE AC CIRCUIT THEORY

Most of the analysis techniques for DC circuits are true in this new "phasor" form of steady-state AC circuits.

- Impedances in series add: $Z_T = Z_1 + Z_2 + \cdots + Z_n$
- Admittances in parallel add: $Y_T = Y_1 + Y_2 + \cdots + Y_n$
- Superposition: The total current in any part of a linear circuit equals the algebraic sum of the currents produced by each source separately
- Voltage law: The net voltage around any closed loop must equal zero
- Current law: The sum of currents at any point must equal zero (the current flowing into the node equals the current flowing out of the node)

The superposition theorem can be used to analyse harmonics. If you have a source that is made up of two sinusoids such as:

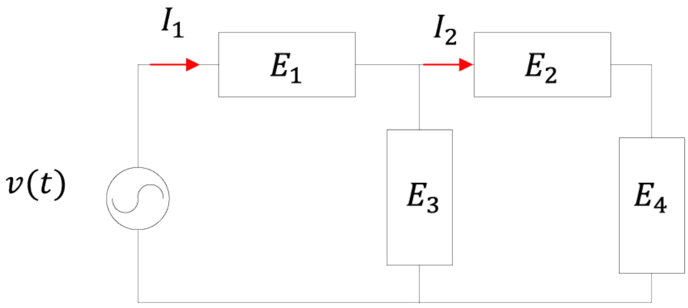
$$v(t) = V_1 \cos(\omega_1 t + \theta_{v1}) + V_2 \cos(\omega_2 t + \theta_{v2})$$

You can analyse the effect of each sinusoid separately. When doing this you have to be careful that you do not combine your sinusoids from each part (the phasors are different frequencies and thus cannot be combined).

Impedance questions

1. Consider the circuit below, where:

$$E_1 = 4\Omega, E_2 = 2\Omega, E_3 = j2\Omega, E_4 = -j2\Omega$$



What is the equivalent impedance of the circuit as seen by the voltage source?

2. A circuit consists of an inductor, L=1 mH, and a resistor, R= 1 Ω, in series. A 50 Hz AC current with an RMS value of 100 A is passed through the series R-L connection.
   a. What is the inductance of each component?
   b. What is the total inductance?
   c. What is the total voltage drop? $V_{Total} = 104.81\angle 17.4°$
   d. Draw the phasor diagram clearly showing the angle between voltage and current.

3. A circuit consists of an inductor, L=1 mH, and a resistor, R= 1 Ω, in series with a voltage source that consists of the sum of two sinusoids of different frequencies

$$v(t) = 100\cos 100\pi t + 20\cos 200\pi t$$

Use phasor analysis and superposition to find the time domain voltage across R & L.

AC POWER

The power in any instant is defined as the product of the voltage and current in that instant. Thus we have:

$$p(t) = v(t) + i(t)$$

$$\mathrm{p}(t) = \sqrt{2}V\cos(\omega t + \theta_v) \times \sqrt{2}I\cos(\omega t + \theta_i)$$

We can simplify this as[3]:

$$\mathrm{p}(t) = P + P\cos(2\omega t) - Q\sin(2\omega t)$$

Where:

$$P = VI\cos(\varphi)$$

$$Q = VI\sin(\varphi)$$

$$\varphi = (\theta_v - \theta_i)$$

Or with some further derivation as:

$$\mathrm{p}(t) = P + |S|\cos(2\omega t + \varphi)$$

Where:

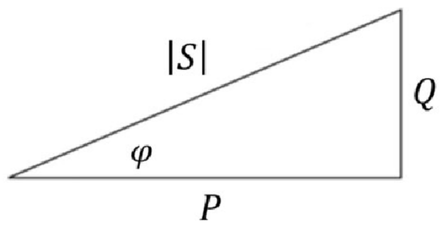
$$|S| = VI$$

There are a few important factors to note about this:

- The instantaneous power in an AC circuit oscillates at *twice* the frequency of the voltage/current.
- The average power in an AC circuit is P, which is defined as $VI\cos(\varphi)$.
- The maximum instantaneous power in an AC circuit is when then cosine is 1, giving a value of $p_{max} = P + |S|$.

Power Triangle

Given the definitions of $P, Q, |S|, \varphi$ we can draw these on a right angled triangle, called the power triangle, to show their relationships. We can do this because we know that: P is exactly 90° out of phase with Q, this defines two sides of the triangle; we also know that $\sqrt{P^2 + Q^2} = |S|$ (see the derivation of AC power), giving us the hypotenuse; and the angle between P and $|S|$ is φ.



We can see the triangle above as a phasor (given the symbol $\boldsymbol{S}$). The value of this phasor is:

$$\boldsymbol{S} = P + jQ = |S|\angle\varphi$$

[3] See the full derivation at the end of the document.

As you might expect each of the symbols is named. What you might not expect is that they have their own units, despite the fact that they all represent power. We do this just to distinguish between these various quantities (even though watts are defined as volts times amps). The definitions are given below:

Symbol	Name	Units
P	Real / Average power	Watts, W
Q	Reactive power	Volt-amp-reactive, VAR
$\|S\|$	Apparent power	Volt-amp, VA
$\boldsymbol{S}$	Complex power	Volt-amp, VA
φ	Phase / power-factor angle	degrees or radians
pf	Power factor	unitless

1. WHAT DOES THE REACTIVE POWER ACTUALLY MEAN

We now have 4 different 'types' of power but what does that actually mean in reality?

Any resistor will cause energy to exit the system; normally this happens by the resistor heating up and the heat dissipating on to other components that are not part of our analysis. This is the real power; it is the power that is actually used to do any useful work (heating, moving things, producing light, etc.).

Ideal (perfect) inductors and capacitors do not consume any energy (on average). Half of the time they are charging their electric/magnetic files and half the time they are discharging them.

Let's say you have an inductor and resistor at the end of a long line, powered by a 50Hz AC generator. The inductor will be charging then discharging its magnetic field every 0.02 sec (1/50). For it to charge, some power will flow down the line into the inductor and be stored in the magnetic field. For it to be discharged the power will flow back down the line to the generator. This is in addition to the power that has to flow down the line to heat up the resistor.

If you measured the peak instantaneous power that flows down the line it would be the sum of the power going into charging the magnetic field of the inductor PLUS the power heating up the resistor. The power going into the resistor is "lost" (leaves the circuit, as heat in this case), but the power going into the inductor just ends up cycling back and forth from generator to inductor and back every 0.2 secs. So we have some power that is cycling back and forth (which we call reactive power) and some that leaves the circuit (real power).

This has a number of knock on effects:

- Power system elements, such as transformers, must be rated according to the apparent power that they are able to supply (not the real power).
- Some real power will be lost in the transmission lines because of the power cycling back and forth to and from loads and generators.
- We can cancel out the effects of an inductive load by adding a capacitor locally. In this case the power will still cycle into and out of the inductor but it will be matched by the capacitor doing the same thing 180° out of phase.

2. POWER FACTOR

Thus, the ratio of real and reactive power is an important thing to consider, this relates to the angle between voltage and current (the power factor angle, φ) as follows:

$$\varphi = \tan^{-1}\left(\frac{Q}{P}\right) = (\theta_v - \theta_i)$$

But, in power systems, we normally refer to the power factor (pf), this is defined as:

$$\text{pf} = \cos\varphi$$

Thus the power factor is a number between 0 and 1, representing the reactive power of a system, relative to the real power. A power factor of 1 means that the voltage and current are in phase, thus there is no power cycling back and forth, this means either the load is purely resistive, or the inductance and capacitance have exactly cancelled out.

As $\cos(x)$ is symmetric about 0 we use the phrase *leading* and *lagging* to distinguish between an inductive and a capacitive load.

Component	**Power factor angle** (φ)	**Power factor** (pf)
Resistor	$\varphi = 0$	1
Inductor	$\varphi = 90°$	0 lag
Capacitor	$\varphi = -90°$	0 lead
Inductive (inductor + resistor)	$\varphi > 0$	e.g. 0.9 lag
Capacitive (capacitor + resistor)	$\varphi < 0$	e.g. 0.9 lead

- Normally loads are inductive (e.g. transformers and motors contain large coils of wire, modelled as resistors and inductors)
- Systems are often designed to have a power factor of 0.9 or better. To change a load to meet a certain power factor is called power factor correction.

3. VARs

Because most loads are inductive, power factors are normally lagging. This means that when extra inductive load is added it generally increases the total reactive power, in other words increases VARs (if you had a capacitive load adding an inductor would reduce VARs, but this generally doesn't happen). Conversely, if you add a capacitor to a load it will generally reduce the magnitude of the reactive power, in other words reduced VARs. Thus it is common to say that capacitive loads generate VARs, and inductive loads consume VARs (in the same way that adding a resistive load caused the real power to increase), thus we say a resistive load consumes watts.

AC Power questions

1. A sinusoidal voltage source with maximum amplitude of 625 V is applied to a 50 Ω resistor. What is the power delivered to the resistor?
2. An electrical load is supplied at 240 Vrms and absorbs 8 kW at 0.8 power factor lagging. Calculate the complex power and the impedance of the load.
3. What is the peak value of the instantaneous power for a load that consumes P = 100 watts and Q = 50 voltampere-reactive?
4. An industrial load is constituted of 50 kW of space heating and of 130 kVA of motoring load with a power factor of 0.60 lagging. The rated voltage is 600 V.
   a. Determine the net current and the global power factor of the plant.

$$I = 275A$$

$$pf = 0.776\ Lagging$$

   b. Determine the capacitance value which can correct the power factor of the plant to unity. The frequency is 60 Hz.

$$C = 456\ \mu F$$

   c. Determine the capacitance value which can correct the power factor of the plant so that it complies with the utility power factor requirement of 0.95. The frequency is 60 Hz.

$$C = 766\ \mu F$$

DERIVATION OF INSTANTANEOUS POWER FOR SINUSOIDAL SOURCE

By definition:

$$p(t) = v(t) \cdot i(t)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

Therefore:

$$p(t) = V_m \cos(\omega t + \theta_v) \cdot I_m \cos(\omega t + \theta_i)$$

Shift the reference and gather terms[4]:

$$p(t) = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos(\omega t)$$

Let (by definition):

$$\varphi = \theta_v - \theta_i$$

Cosine product law:

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

Where: $\alpha = \omega t + \varphi, \beta = \omega t$

$$p(t) = V_m I_m \left[\frac{1}{2}\cos\big((\omega t + \varphi) - \omega t\big) + \frac{1}{2}\cos(\omega t + \varphi + \omega t)\right]$$

Simplify:

$$p(t) = {}^{V_m I_m}\!/_2 \, [\cos(\varphi) + \cos(2\omega t + \varphi)]$$

Using RMS values (only true for sinusoidal waves, which these are)

RMS value for sine waves:

$$V_m \, cos(\omega t + \theta) \equiv \sqrt{2} V_{rms} \, cos(\omega t + \theta)$$

$${}^{V_m I_m}\!/_2 = {}^{V_m I_m}\!\Big/_{\sqrt{2}\sqrt{2}} = V_{rms} I_{rms}$$

$$p(t) = V_{rms} I_{rms} [\cos(\varphi) + \cos(2\omega t + \varphi)]$$

[4] All we are doing here is saying $t = t - \theta_i$. I realize that this should strictly give the equation for $p(t - \theta_i)$ but we can choose our reference time and it makes no difference to the analysis.

Cosine angle-sum rule:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Using cosine angle-sum to separate the rightmost cosine, where: $\alpha = 2\omega t, \beta = \varphi$

$$\mathrm{p}(t) = V_{rms}I_{rms}[\cos(\varphi) + \cos(2\omega t)\cos(\varphi) - \sin(2\omega t)\sin(\varphi)]$$

$$\mathrm{p}(t) = V_{rms}I_{rms}\cos(\varphi) + V_{rms}I_{rms}\cos(\varphi)\cos(2\omega t) - V_{rms}I_{rms}\sin(\varphi)\sin(2\omega t)$$

Let (by definition):

$$P = V_{rms}I_{rms}\cos(\varphi)$$

$$Q = V_{rms}I_{rms}\sin(\varphi)$$

Then:

$$\mathrm{p}(t) = P + P\cos(2\omega t) - Q\sin(2\omega t)$$

Harmonic addition theorem:

$$a\cos(x) + b\sin(x) = \sqrt{a^2 + b^2}\cos\left(x + \tan^{-1}\left(\frac{-b}{a}\right)\right)$$

Where $a = P, b = -Q, x = 2\omega t$

$$\mathrm{p}(t) = P + \sqrt{P^2 + (-Q)^2}\cos\left(2\omega t + \tan^{-1}\left(\frac{Q}{P}\right)\right)$$

$$\frac{Q}{P} = \frac{V_{rms}I_{rms}\sin(\varphi)}{V_{rms}I_{rms}\cos(\varphi)} = \tan(\varphi)$$

$$\mathrm{p}(t) = P + \sqrt{P^2 + Q^2}\cos(2\omega t + \varphi)$$

$$\sqrt{P^2 + Q^2} = \sqrt{(V_{rms}I_{rms}\cos(\varphi))^2 + (V_{rms}I_{rms}\sin(\varphi))^2}$$

Factor out $V_{rms}I_{rms}$:

$$\sqrt{P^2 + Q^2} = \sqrt{(V_{rms}I_{rms})^2 \times (\cos(\varphi)^2 + \sin(\varphi)^2)}$$

Trigonometric identity:

$$\cos(\theta)^2 + \sin(\theta)^2 = 1$$

$$\sqrt{P^2 + Q^2} = \sqrt{(V_{rms}I_{rms})^2} = V_{rms}I_{rms}$$

Let (by definition):

$$|S| = V_{rms}I_{rms}$$

Therefore:

$$\mathrm{p}(t) = P + |S|\cos(2\omega t + \varphi)$$

P is known as the average power, written in form above it is clear why. The cosine term averages to 0 leaving the average to be P.

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SUSTAINABLE ELECTRICAL POWER SYSTEMS ENGINEERING

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Royal Charter Number: RC000797