

**Department of Mathematics**  
**Postgraduate Research Projects Booklet**  
**2020-2021**

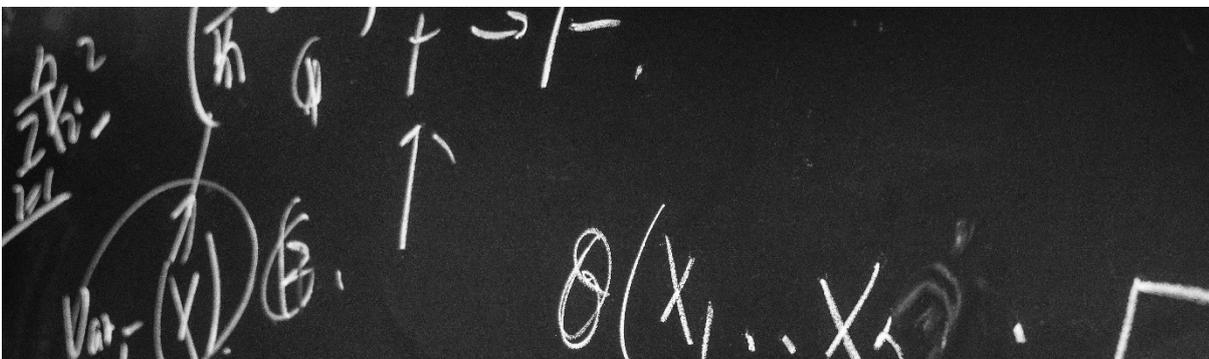


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## Applied Mathematics and Numerical Analysis

Below are the projects that the Department of Mathematics offers for PhD study within the areas of Applied Mathematics and Numerical analysis.

For further information, please feel free to contact [pgr-maths@manchester.ac.uk](mailto:pgr-maths@manchester.ac.uk).

**Title:** Mathematics & Meteorites

**Group:** Industrial and Applied Mathematics

**Supervisor:** Dr Geoff Evatt ([Geoffrey.Evatt@manchester.ac.uk](mailto:Geoffrey.Evatt@manchester.ac.uk))

**Description:** Man-kind has long been fascinated by the falling of material from space. This material gives us insight into the composition of the early solar system and the remaining planets within. Huge efforts across the globe are placed into the analysis of such material and their parent bodies, plus how it is transported and delivered around the solar system. However the use of applied mathematics (very broadly defined), presents a prism for studying this material from a novel vantage -an example being the 'The Lost Meteorites of Antarctica Project' ([www.ukantarcticmeteorites.com](http://www.ukantarcticmeteorites.com)). Other applications for mathematics include fireball camera network analysis (e.g. predicting what an observed fireball actually was), crater counting studies within the solar system, and collection statistics from Antarctica. Having now established ourselves as a leader in such studies, we seek applications from PhD students will a strong background in mathematics and firm interdisciplinary mindset, who wish to push boundaries in this exciting area of space and planetary science.

**Title:** Fast and Reliable Algorithms for High Performance Numerical Linear Algebra

**Group:** Numerical Analysis and Scientific Computing

**Supervisor:** Nick Higham ([Nick.Higham@manchester.ac.uk](mailto:Nick.Higham@manchester.ac.uk))

**Description:** The project focuses on developing a new generation of numerical linear algebra algorithms that exploit current and future computers. The algorithms need to be fast and to be accompanied by rigorous error analysis to guarantee their reliability, even for the largest and most difficult problems. The target problems will be drawn from linear equations, linear least squares problems, eigenvalue problems, the singular value decomposition, and matrix function evaluation. These are the innermost kernels in many

scientific and engineering applications---in particular, in data science and in machine learning---so it is essential that they are fast, accurate, and reliable.

A key aspect of this work is the exploitation of variable precision arithmetic. Low precision arithmetic is now available in hardware and is increasingly being used in machine learning and scientific computing more generally because of its speed, but its limited precision and narrow range require careful treatment. High precision (quadruple precision and above) is available in software and may be used in small amounts to speed up or stabilize an algorithm.

A strong background in numerical linear algebra and programming skills in MATLAB or a high level language are essential.

This project provides the opportunity to be part of a large and vibrant numerical linear algebra group that has several strongly committed industrial partners (see <https://nla-group.org/>).

**Funding Notes:** Open to all. Funding is available and would provide fees and maintenance at RCUK level for home/EU students, or a fees-only bursary for overseas students.

**Title:** Fluid-structure interaction effects in the sedimentation of thin elastic sheets

**Group:** Industrial and Applied Mathematics/ Continuum Mechanics/Numerical Analysis and Scientific Computing

**Supervisors:**

- Professor Matthias Heil (Department of Mathematics)  
([Matthias.Heil@manchester.ac.uk](mailto:Matthias.Heil@manchester.ac.uk))
- Professor Anne Juel (Department of Physics and Astronomy)  
([Anne.Juel@manchester.ac.uk](mailto:Anne.Juel@manchester.ac.uk))
- Dr Draga Pihler-Puzovic (Department of Physics and Astronomy)  
([draga.pihler-puzovic@manchester.ac.uk](mailto:draga.pihler-puzovic@manchester.ac.uk))

**Description**

There is much current interest in so-called two-dimensional materials because of their unusual and attractive mechanical and electrical properties. Much of their processing is performed in a fluid environment, e.g., during the size selection of dispersed graphene flakes by centrifugation, or their deposition by ink-jet printing. The flakes' large aspect ratio implies that despite their impressive in-plane stiffness they have a very small bending stiffness and are therefore easily deformed by the traction that the surrounding

fluid exerts on them. The resulting strong fluid-structure interaction affects not only the dynamics of individual flakes but also their collective behaviour.

The aim of this project is to perform a systematic study of the behaviour of thin elastic sheets in a viscous fluid. Specifically, we wish to establish how the flow-induced deformation affects the sedimentation of such sheets, paying particular attention to

- the effect of the sheets' aspect ratio; long narrow sheets are likely to behave in a manner similar to elastic rods: at which point does their finite aspect ratio become significant?

- the effect of wrinkling instabilities and the development of symmetry-breaking frustrated patterns: how do they arise in sheets of canonical shapes (circular, rectangular, polygonal,...) and how do they affect the sheets' sedimentation?

The focus of this specific project is on computational/semi-analytical approaches and would suit a student with a good background in Applied Mathematics (especially fluid and solid mechanics) and Scientific Computing. There is an opportunity for hands-on involvement in an associated experimental study in the School of Physics and Astronomy.

**Main Supervisor:** Professor Matthias Heil (Department of Mathematics)

**Co-Supervisor:** Professor Anne Juel (Department of Physics and Astronomy)

Dr Draga Pihler-Puzovic (Department of Physics and Astronomy)

**Other associated project areas:** Physics

**Funding:** Funding is available and would provide fees and maintenance at RCUK level for home/EU students, or a fees-only bursary for overseas students.

**Deadline:** Applications are accepted at any time until the position is filled.

**Title:** Complex deformations of biological soft tissues

**Group:** Continuum Mechanics

**Supervisors:**

- Andrew Hazel ([Andrew.Hazel@manchester.ac.uk](mailto:Andrew.Hazel@manchester.ac.uk))
- Tom Shearer ([Tom.Shearer@manchester.ac.uk](mailto:Tom.Shearer@manchester.ac.uk))

**Description:** The answers to many open questions in medicine depend on understanding the mechanical behaviour of biological soft tissues. For example, which tendon is most appropriate to replace the anterior cruciate ligament in reconstruction surgery? what

causes the onset of aneurysms in the aorta? and how does the mechanics of the bladder wall affect afferent nerve firing? Current work at The University of Manchester seeks to understand how the microstructure of a biological soft tissue affects its macroscale mechanical properties. Most of the work to date has focused on simple deformations (e.g. longitudinal extension under tension) for which analytical solutions can be found. However, the geometry and deformation of many soft tissues in vivo is sufficiently complex to prohibit analytical solutions.

In this project, we will use our “in house” finite element software oomph-lib to investigate complex deformations of biological soft tissues. The work will require development and implementation of novel strain energy functions as well as formulation of non-standard problems in solid mechanics. The project is likely to appeal to students with an interest in continuum mechanics, computational mathematics and interdisciplinary science.

**Title:** Instabilities of buoyancy driven flows in a confined environment

**Group:** Continuum Mechanics/ Industrial and Applied Mathematics

**Supervisor:** Dr Julien Landel ([julien.landel@manchester.ac.uk](mailto:julien.landel@manchester.ac.uk))

**Description:** Buoyancy driven flows are important in many geophysical and environmental applications: from natural ventilation in buildings to the evolution of cloud particles after a volcanic eruption or an explosion. Much work has been done on these flows in both turbulent and laminar flow regimes. However, the impact of lateral confinement on the flow has not received as much attention. One of the distinctive features is the development of an instability due to lateral shear. This effect can have important consequences for the large scale dynamics as well as the small scale mixing and dispersion properties of these flows when carrying particles or other tracers.

In this project, we will explore experimentally and theoretically the origin of the instability for various fundamental buoyancy-driven flows such as fountains and thermals. New experimental techniques, based on novel experimental design and imaging techniques, recently developed in our laboratory have allowed to probe further into the complex dynamics of these flows. The goal of this project is to study the phenomenology of the flow in order to determine the source of the instability. We will also analyse how the instability affects mixing and dispersion of active or passive tracers.

The project is suitable for an enthusiastic and creative candidate who has some experience in laboratory experimentation and good knowledge in fluid mechanics. Some knowledge in imaging analysis technique is desired but not necessary.

**Funding Notes:** Funding is available and would provide fees and maintenance at RCUK level for home/EU students, or a fees-only bursary for overseas students. Competitive bursaries are also available for overseas students to fully cover both fees and maintenance at RCUK level.

**Title:** Thermo-visco-acoustic metamaterials for underwater applications

**Group:** Continuum Mechanics/ Industrial and Applied Mathematics

**Supervisor:** Professor William J Parnell ([William.J.Parnell@manchester.ac.uk](mailto:William.J.Parnell@manchester.ac.uk))

**Description:** The ability to control underwater noise has been of practical interest for decades. Such noise, radiating from e.g. offshore wind farms, turbines, and merchant vessels, frequently needs to be attenuated artificially given the close proximity of its generation to sensitive marine environments for example.

Over the last century a number of materials have been designed to assist with underwater noise attenuation. However, recently there has been an explosion of interest in the topic of acoustic metamaterials and metasurfaces. Such media have special microstructures, designed to provide overall (dynamic) material properties that natural materials can never hope to attain and lead to the potential of negative refraction, wave redirection and the holy grail of cloaking. Many of the mechanisms to create these artificial materials rely on the notion of resonance, which in turn gives rise to the possibility of low frequency sound attenuation. This is extremely difficult to achieve with normal materials.

The mechanisms of sound attenuation, i.e. thermal and viscous, have not yet been properly understood for the many metamaterials under study, particularly in an underwater context. The aim of this project is to study this aspect via mathematical analysis and then to optimize designs in order to design and employ metamaterials for use in underwater noise reduction applications. Although there has been some initial interest over the last few years in the “in-air” context, the parameter regime underwater gives rise to new effects that need to be explored and understood thoroughly.

Initially canonical geometries such as simple apertures and infinite and semi-infinite ducts shall be considered before moving on to more complex, realistic scenarios and geometries where resonance plays a key role.

Mathematical modelling using the method of matched asymptotics shall be employed. This is ideally suited to the scenarios considered given the low frequency regime. Comparisons shall be drawn with direct numerical simulations using finite element methods in e.g. COMSOL.

**Title:** Fluid Mechanics of Cleaning and Decontamination

**Group:** Continuum Mechanics/ Industrial and Applied Mathematics

**Supervisor:** Dr Julien Landel ([julien.landel@manchester.ac.uk](mailto:julien.landel@manchester.ac.uk))

**Description:** Cleaning and decontamination processes are important in many applications: from the daily chores of doing the dishes (with or without a dishwasher), to ensuring clean hygiene in hospitals, the food industry, or pharmaceutical companies. Although a lot of research has been done in chemistry and chemical engineering to improve detergents and cleaning devices, much less work has been done on the modelling of the underlying physical and chemical processes. In some cleaning applications, such as the neutralisation of toxic chemicals after a spill, it is crucial to avoid using strong mechanical forces in order to prevent the dispersion of the toxic material in the environment. Instead, a localised dissolution process, aided by chemical reactions neutralising the material, is used. This PhD project will investigate the advection, diffusion and reaction processes involved in this scenario.

Through a combination of experiments and modelling work the student will study the influence of flow properties: such as the Reynolds number and the Péclet number; geometry: whether the material is attached to a permeable or impermeable surface; and chemical properties such as solubility, reactivity and diffusivity.

This project is directly motivated by industrial applications and will suit candidates interested in using mathematical approaches to solve real challenges. Suitable candidates should have experience in the lab or a keen interest to support theoretical work in fluid dynamics by experimental evidence.

**Reference:** Landel, Thomas, McEvoy & Dalziel (2016). Convective mass transfer from a submerged drop in a thin falling film, *Journal of Fluid Mechanics*, 789: 630.

**Title:** Evaporation and deposition patterns in non-axisymmetric droplets

**Group:** Continuum Mechanics

**Supervisors:**

- Dr Alice Thompson ([alice.thompson@manchester.ac.uk](mailto:alice.thompson@manchester.ac.uk))
- Professor Andrew Hazel ([andrew.hazel@manchester.ac.uk](mailto:andrew.hazel@manchester.ac.uk))

**Description:** Inkjet printing can be used as a flexible method to create micro-scale electronic circuits and other electronic components. These components require the deposition of conductive particles or polymers in particular locations on a substrate.

However, inkjet printing involves liquid drops, so the particles are first embedded in a carrier fluid for printing. Once on the substrate, these liquid drops can interact and spread, before the carrier fluid eventually evaporates, leaving a deposit of the original particles. We have previously developed efficient simple models for the first stage of this process: how liquid drops interact and spread on the substrate. However, the overall outcome of the process is the deposition pattern left behind, which can depend subtly on the shape and properties of the deposited particles. Our aim here is to use a range of mathematical models to predict the deposition patterns, and to consider to what extent the printing process could be optimised to control the deposition distributions. This project will mainly involve mathematical modelling and computational methods, with scope for comparison with experiments.

**Title:** Turbulent particle-laden jets

**Group:** Continuum Mechanics/Industrial and Applied Mathematics

**Supervisors**

- Dr Julien Landel ([julien.landel@manchester.ac.uk](mailto:julien.landel@manchester.ac.uk))
- Dr Rich Hewitt ([richard.hewitt@manchester.ac.uk](mailto:richard.hewitt@manchester.ac.uk))

**Description:** Turbulent particle-laden jets are relevant to many geophysical and industrial applications: from volcanic eruptions, to sediment resuspension, fluidisation processes and chemical reactors. Much work has been done on the dilute regime of these two-phase flows, where the particles have a small impact on the fluid and can often be considered as passive tracers. In this experimental project, we focus on the poorly understood dense regime, where the coupling between the solid particles and the fluid is more complex.

Many fundamental questions, of high relevance to the applications mentioned above, are still unresolved. This project will explore the impact of the particle density on turbulent entrainment processes. Entrainment processes during an explosive volcanic eruption have a considerable impact on the extent of the damages. They determine whether the eruption will collapse and form a pyroclastic flow, with local implications, or whether the eruption column will rise and form an ash cloud spreading over extended regions, such as in the case of the 2010 eruption of the Icelandic volcano Eyjafjallajökul. This project will also explore the effect on mixing processes, which are very important for instance in chemical reactors where the efficiency of the reaction depends strongly on the efficiency of the mixing.

These dense particle-laden jets are still poorly understood due to the considerable challenges faced analytically and numerically. Technical difficulties have also prevented

progress on the experimental side for a long time. New experimental techniques, based on novel experimental design and imaging techniques, recently developed in the laboratory have allowed to probe much further into the complex dynamics of these dense particle laden jet. The main goal of this project is to pursue the development of these techniques in order to address the questions on entrainment and mixing described above.

The project is suitable for an enthusiastic and creative candidate who has some experience in experimentation and good knowledge in fluid mechanics. Some knowledge in imaging analysis technique is desired but not necessary. The motivation and readiness of the candidate to learn new techniques and develop them to explore fundamental scientific questions will be key to the success of this project.

**Title:** Efficient Uncertainty Quantification for PDEs with Random Data

**Group:** Industrial and Applied Mathematics

**Supervisor:** Dr. Catherine Powell ([c.powell@manchester.ac.uk](mailto:c.powell@manchester.ac.uk))

**Description:** Uncertainty Quantification (UQ) is the science of accounting for uncertainty in mathematical models. Research in this area has undergone rapid growth in the last few years and is currently considered a 'hot topic'. This growth has been driven by the need for scientists in today's world to provide decision makers with ever more accurate and reliable predictions that are based on results obtained from mathematical models.

Many physical processes such as fluid flows are governed by partial differential equations (PDEs). In practical applications in the real world, it is unlikely that all the inputs (boundary conditions, geometry, coefficients) for the chosen PDE model will be known. One possibility is to model the quantities that we don't know as random variables. Solving these problems is not always hard in theory but solving them efficiently in practice is a massive challenge.

I am interested in working with students who want to develop numerical analysis and numerical methods (e.g. solvers, error estimators) for solving partial differential equations with uncertain inputs (stochastic PDEs). I welcome any enquiries to work in this area. Specific projects could be theoretical or computational, according to the strengths of the student.

Projects on this topic would suit students who have taken undergraduate courses in numerical analysis and applied mathematics who have a keen interest in computational mathematics and developing practical algorithms. Some prior programming experience is essential.

**Background reference:**

An Introduction to Computational Stochastic PDEs (Cambridge Texts in Applied Mathematics), G. J. Lord, C.E. Powell and T. Shardlow, 2014.

**Title:** Mathematical theory of diffraction

**Group:** Continuum Mechanics

**Supervisor:** Raphael Assier ([raphael.assier@manchester.ac.uk](mailto:raphael.assier@manchester.ac.uk))

**Description:** There is a long history of mathematicians working on canonical diffraction (or scattering) problems. The mathematical theory of diffraction probably started with the work of Sommerfeld at the end of the 19th century and his famous solution to the diffraction of acoustic waves by a solid half-plane. Since, some very ingenious mathematical methods have been developed to tackle such problems. One of the most famous being the Wiener-Hopf technique.

However, despite tremendous efforts in this field, some canonical problems remain open, in the sense that no clear analytical solution is available for them.

One of this problem is the quarter-plane problem, the problem of diffraction of acoustic waves by a solid quarter-plane. Thus far, it has not been possible to apply classical methods such as the Wiener-Hopf method successfully in that case, and hence some new mathematical methods need to be developed in order to tackle this problem. This makes it very interesting as it implies that many different types of mathematics can be used and it makes the subject intrinsically multidisciplinary.

Many industrial problems can be linked to the theory of diffraction, for example the noise generated by a jet engine (acoustic waves) or radar detection (electro-magnetic waves) and defect detection in materials (elastic waves).

PhD projects are available in this field.

**References:**

-- R. C. Assier and N. Peake. On the diffraction of acoustic waves by a quarter-plane. *Wave Motion*, 49(1):64-82, 2012

-- R. C. Assier and N. Peake. Precise description of the different far fields encountered in the problem of diffraction of acoustic waves by a quarter-plane. *IMA J. Appl. Math.*, 77(5):605-625, 2012.

**Title:** Fractional differential equations and anomalous transport

**Group:** Continuum Mechanics

**Supervisor:** Sergei Fedotov ([sergei.fedotov@manchester.ac.uk](mailto:sergei.fedotov@manchester.ac.uk))

**Description:** This project is concerned with anomalous transport, which cannot be described by standard calculus. Instead it requires the use of fractional differential equations involving fractional derivatives of non integer order. This is a new, exciting area of research because anomalous transport is a widespread natural phenomenon. Examples include flight of albatross, stock prices, human migration, social networks, transport on fractal geometries, proteins on cell membranes, bacterial motion, and signalling molecules in the brain.

**Title:** Stability and separation in  $R \gg 1$  flows

**Group:** Continuum Mechanics

**Supervisor:** Jitesh S B Gajjar ([j.gajjar@manchester.ac.uk](mailto:j.gajjar@manchester.ac.uk))

**Description:** I have several projects available in the area of high Reynolds number flows, including the study of laminar separation and stability of thin films, cavity flows, break-up of separation bubbles, cross-flow instability. The work can be theoretical, numerical or a mixture of both.

**Title:** Combustion instabilities

**Group:** Continuum Mechanics

**Supervisor:** Raphael Assier ([raphael.assier@manchester.ac.uk](mailto:raphael.assier@manchester.ac.uk))

**Description:** Combustion is essential to energy generation and transport needs, and will remain so for the foreseeable future. Mitigating its impact on the climate and human health, by reducing its associated emissions, is thus a priority. One suggested strategy to reduce NO<sub>x</sub> is to operate combustors at lean conditions. Unfortunately, combustion instability is more likely to occur in the lean regime, and may have catastrophic consequences on the components of combustion chambers, such as vibrations and structural fatigue.

Ramjet engines, rocket engines and in general any type of gas turbine engines may be subject to this detrimental instability. The ability to predict and control the instability is crucial for implementing the lean burn strategy. Combustion instability involves an intricate interplay of several key physical processes, which take place in regions of

different length scales. Due to this multi-scale, multi-physics nature of the problem, direct numerical simulations of realistic combustors are extremely challenging. For this reason, simplified mathematical models capturing qualitatively and quantitatively the main characteristics of combustion instability are essential. In particular, by exploiting the scale disparity, systematic asymptotic analyses may be carried out to derive relevant models on first principles, and to provide guidance for developing reliable and efficient numerical algorithms.

Recent progress have been made in the mathematical modelling of such instabilities, using refining and implementing this model would make a good PhD project.

**Reference:**

R. C. Assier and X. Wu. Linear and weakly nonlinear instability of a premixed curved flame under the influence of its spontaneous acoustic field. *J. Fluid Mech.* (2014), vol. 758, pp. 180-220

**Title:** Microstructural models of the constitutive behaviour of soft tissue

**Group:** Continuum Mechanics

**Supervisors:**

- William J Parnell ([william.parnell@manchester.ac.uk](mailto:william.parnell@manchester.ac.uk))
- Tom Shearer ([tom.shearer@manchester.ac.uk](mailto:tom.shearer@manchester.ac.uk))

**Description:** Soft tissue such as tendon, ligament, skin, and the brain possess complex nonlinear viscoelastic constitutive behaviour which arises due to the intricate microstructures inherent in such materials. The majority of existing models for the constitutive behaviour of soft tissue are phenomenological so that the parameters involved in the model are not derivable from experiments.

In this project the objective is to build models that are based on the microstructure and we will liaise with experimentalists, particularly those in imaging science, in order to ensure that the parameters involved can be directly measured.

This project would suit those with a strong background in continuum mechanics and modelling and although not essential some background knowledge in nonlinear elasticity would be useful.

**Title:** Revenue Management

**Group:** Mathematical Finance and Actuarial Science

**Supervisor:** Paul Johnson ([Paul.Johnson-2@manchester.ac.uk](mailto:Paul.Johnson-2@manchester.ac.uk))

**Description:** By Revenue Management (RM), we mean the process of understanding, forecasting and influencing consumer behaviour in order to maximise a firm's revenues. Put simply, RM is all about selling the right product to the right customer at the right time for the right price. RM originated in the airline industry under the term Yield Management. Today, RM is widely employed in many other major industries, such as hotels, restaurants, car rentals and carparks.

A prospective research student in this area should expect to gain knowledge on Optimisation, Markov Processes, Dynamic programming and HJB equations.

**Title:** Environmental fluid mechanics

**Group:** Continuum Mechanics

**Supervisor:** Dr. R.E. Hewitt ([richard.e.hewitt@manchester.ac.uk](mailto:richard.e.hewitt@manchester.ac.uk))

**Description:** Many problems of environmental significance require the effective prediction of particulate (contaminant) transport in a fluid system (which constitutes a 'two-phase' fluid/particle problem). The primary focus of this project is a suspension of solid particles (dust/ash) in a viscous incompressible fluid. Most practical cases of interest have particles that are typically fractions of a millimetre in size, but still occupy a non-small fraction of the total mixture mass and exist in large numbers. The simultaneous treatment of all individual particles (and the correspondingly complicated fluid domain) is computationally impractical, a state of affairs that will remain for the foreseeable future.

Furthermore, the behaviour of a single particle cannot be solved in isolation of the other particles, owing to particle-particle interactions through the motion of the interstitial fluid, or by direct particle collisions at high concentration levels. In such cases, both phases of the mixture exchange momentum with the other, so that the fluid motion and the particle motion remain coupled together. Furthermore, the presence of bounding surfaces for the fluid mixture can have crucial consequences for the structural and temporal development of the flow and the distribution of suspended material.

This project aims to continue the development of existing macro scale models, in which both phases are treated as co-existing (coupled) continua, through a combination of analytical and computational methods.

**Title:** Plant tissue mechanics

**Group:** Continuum Mechanics

**Supervisor:** Professor Oliver Jensen ([Oliver.Jensen@manchester.ac.uk](mailto:Oliver.Jensen@manchester.ac.uk))

**Description:** Plant growth arises through the coordinated expansion of individual cells, allowing a plant to adapt to its environment to harness light, water and essential nutrients. Growth is driven by the high internal turgor pressure of cells and is regulated by physical and biochemical modifications of plant cell walls. Many features of this immensely complex process remain poorly understood, despite its profound societal and environmental importance. Mathematical models describing the mechanical properties of a growing plant tissue integrate features ranging from molecular interactions within an individual cell wall to the expansion, bending or twisting of a multicellular root or stem. Building on current biological understanding, this project will address the development and analysis of new multiscale models for plant tissues, exploiting a variety of computational and asymptotic techniques.

**Background references:**

1. Dyson, RJ & Jensen, OE (2010) J Fluid Mech 655, 472
2. Dyson, RJ, Band, L & Jensen, OE (2012) J Theor Biol 307, 125
3. Baskin, TI & Jensen, OE (2013) J Exp Bot 64, 4697
4. Dyson, RJ et al. (2014) New Phytologist 202, 1212

**Title:** Numerical Analysis and Computational Methods for Solving PDEs with Uncertainty

**Group:** Numerical Analysis and Scientific Computing

**Supervisor:** Dr. Catherine Powell ([c.powell@manchester.ac.uk](mailto:c.powell@manchester.ac.uk))

**Description:** Uncertainty Quantification (UQ) is the science of accounting for uncertainty in mathematical models. Research in this area has undergone rapid growth in the last few years and is currently considered a 'hot topic'. This growth has been driven by the need for scientists in today's world to provide decision makers with ever more accurate and reliable predictions that are based on results obtained from mathematical models.

Many physical processes such as fluid flows are governed by partial differential equations (PDEs). In practical applications in the real world, it is unlikely that all the inputs (boundary conditions, geometry, coefficients) for the chosen PDE model will be known. One possibility is to model the quantities that we don't know as random

variables. Solving these problems is not always hard in theory but solving them efficiently in practice is a massive challenge.

I am interested in working with students who want to develop numerical analysis and numerical methods for solving partial differential equations with uncertain inputs. I welcome any enquiries to work in this area. Specific projects could be theoretical or computational, according to the strengths of the student.

Projects on this topic would suit students who have taken undergraduate courses in numerical analysis and applied mathematics who have a keen interest in computational mathematics and developing practical algorithms. Some prior programming experience is essential.

**Background reference:**

An Introduction to Computational Stochastic PDEs (Cambridge Texts in Applied Mathematics), G. J. Lord, C.E. Powell and T. Shardlow, 2014.

**Title:** Interactions between rocks and ice

**Group:** Continuum Mechanics

**Supervisor:** Professor Matthias Heil ([M.Heil@maths.man.ac.uk](mailto:M.Heil@maths.man.ac.uk))

**Description:** Many glaciers are covered by a debris layer whose presence has multiple, competing effects on the glacier's melt rate. The debris layer shields the ice from incoming solar radiation and thus reduces its melt rate. However, since the albedo of the debris layer is much smaller than that of the ice, the debris layer is heated up very rapidly by the solar radiation, an effect that is likely to increase the melt rate.

The project aims to develop theoretical/computational models to study how solid objects (rocks) which are placed on (or embedded in) an ice layer affect the ice's melt rate. The work will employ (and contribute to) the object-oriented multi-physics finite-element library oomph-lib, developed by M. Heil and A.L. Hazel and their collaborators, and available as open source software at <http://www.oomph-lib.org>.

The project would suit students with an interest in mathematical modelling, continuum mechanics and scientific computing and will be performed in close collaborations with Glaciologists at the University of Sheffield and the Bavarian Academy of Science.

**Title:** The Dynamics of Debris Flows

**Group:** Continuum Mechanics

**Supervisors:** Chris Johnson ([chris.johnson@manchester.ac.uk](mailto:chris.johnson@manchester.ac.uk))

Nico Gray ([nico.gray@manchester.ac.uk](mailto:nico.gray@manchester.ac.uk))

**Description:** Debris flows are rapid avalanches of rock and water, which are triggered on mountainsides when erodible sediment is destabilised by heavy rainfall or snowmelt. These flows cause loss of life and infrastructure across the world, but many of the physical mechanisms underlying their motion remain poorly understood. Among these are the processes of erosion and deposition, which dramatically increase and decrease the mass of a debris flow, and polydisperse grain size segregation, which separates the large rocks in a flow from a water-saturated suspension of smaller particles. This project focuses on developing theoretical models for these processes that are constrained by observations of natural debris flows and flow deposits.

**References:**

C. G. Johnson et al., Grain-size segregation and levee formation in geophysical mass flows J. Geophys. Res. 117, F01032, (2012)

[https://www.maths.manchester.ac.uk/~cjohnson/papers/johnson\\_jgr\\_2012.pdf](https://www.maths.manchester.ac.uk/~cjohnson/papers/johnson_jgr_2012.pdf)

F. M. Rocha et al., Self-channelisation and levee formation in monodisperse granular flows, J. Fluid Mech. 876, (2019)

<https://doi.org/10.1017/jfm.2019.518>

S. Viroulet et al., The kinematics of bidisperse granular roll waves, J. Fluid Mech. 848, 836–875, (2018)

<https://doi.org/10.1017/jfm.2018.348>

**Title:** Microscale fluid dynamics around coral surfaces

**Group:** Continuum Mechanics/Applied Mathematics

**Supervisor:** Professor Peter Duck ([peter.duck@manchester.ac.uk](mailto:peter.duck@manchester.ac.uk))

The University of Melbourne: Douglas Brumley, John Sader, and Linda Blackall.

**Description:** This fully funded project will develop a new biophysical approach to coral studies, integrating mathematical modelling, numerical simulations and lab-on-chip experiments. Although the Great Barrier Reef spans more than 2,300km, fundamental processes driving the health and function of corals such as nutrient uptake, oxygen

exchange and bacterial recruitment occur at sub-millimetre scales. Despite their importance, we still largely lack a quantitative understanding of the governing transport processes.

The PhD project will investigate three-dimensional fluid flows generated by corals and their physical/ecological consequences. Through an iterative approach involving mathematical modelling and experiments, the project will develop a mechanistic framework for understanding microscale biophysical processes at the coral surface. This will inform efforts in coral conservation and management and be broadly applicable to range of other problems in biology.

Throughout their candidature, the PhD student will work in an international cross-disciplinary research team, gaining a range of skills in fluid dynamics, continuum modelling, numerical analysis (including scientific computing), microfluidics, microscopy, and image processing. During their candidature, the student will spend at least 12 months at each of The Universities of Melbourne and Manchester, working with leading research groups in applied mathematics, fluid dynamics and coral biology.

**Funding:** Funding for the successful candidate will include (1) tuition fees at Melbourne and Manchester; (2) The Melbourne Research Scholarship (100% fee remission and up to \$110,000 AUD); and (3) Travel allowance.

**Contact:** Informal enquiries can be made to Dr. Douglas Brumley with a copy of the Applicant's curriculum vitae. Applications will be considered on a rolling basis, with a final deadline of 22 November 2019.

<https://mmgrg.research.unimelb.edu.au/2019/10/17/microscale-fluid-dynamics-around-coral-surfaces/>

**How to Apply:** <https://mmgrg.research.unimelb.edu.au/2019/10/03/how-to-apply/>

**Title:** Generalized Flame Balls and their Stability

**Group:** Continuum Mechanics

**Supervisor:** Dr Joel Daou ([joel.daou@manchester.ac.uk](mailto:joel.daou@manchester.ac.uk))

**Description:** Flame balls are balls of burnt gas in a reactive mixture, which constitute stationary solutions to non-linear Poisson's equations. These were first described by the famous Russian physicist Zeldovich (the father of Combustion Theory) about 70 years ago. The fact that these solutions are typically unstable provides a powerful fundamental criterion for successful ignition, i.e. determines the minimum energy (of the spark) required to generate propagating flames. Several projects are available to extend the study of these fascinating flames (mainly their existence and stability) to take

into account realistic effects such as the presence of flow-field, non-uniformity of the reactive mixture, proximity of walls, etc.

**Methodology:** The approach will typically adopt a combination of analytical techniques (asymptotic methods) and/or numerical techniques (solution of ODEs or PDEs), depending on the preference of the candidate.

**Title:** Laminar aspects of turbulent combustion/ Flame propagation in a multi-scale flow

**Group:** Continuum Mechanics

**Supervisor:** Dr Joel Daou ([joel.daou@manchester.ac.uk](mailto:joel.daou@manchester.ac.uk))

**Description:** The idea is to ask if the fundamental questions of turbulent combustion can be answered for simple laminar flows. Since the answer is often no, we shall formulate and study problems to answer these questions in simpler laminar-flow situations. An exciting topic!

**Methodology:** The approach will typically adopt a combination of analytical techniques (asymptotic methods) and/or numerical techniques (solution of ODEs or PDEs), depending on the preference of the candidate.

**Title:** Taylor dispersion and hydrodynamic lubrication theory in premixed combustion

**Group:** Continuum Mechanics

**Supervisor:** Dr Joel Daou ([joel.daou@manchester.ac.uk](mailto:joel.daou@manchester.ac.uk))

**Description:** In 1953, the British physicist G.I. Taylor published an influential paper describing the enhancement of diffusion processes by a (shear) flow, a phenomenon later termed Taylor dispersion. This has generated to date thousands of publications in various areas involving transport phenomena, none of which, surprisingly, in the field of combustion. In 1940, the German chemist G. Damköhler postulated two hypotheses which have largely shaped current views on the propagation of premixed flames in turbulent flow fields. The project consists of pioneering investigations linking Taylor dispersion and Damköhler's hypotheses, and is expected to provide significant insight into turbulent combustion. The work will be carried out in the framework of lubrication theory, generalized to combustion situations, and will include interesting stability problem such as the Saffman-Taylor instability in a reactive mixture.

**Methodology:** The approach will typically adopt a combination of analytical techniques (asymptotic methods) and/or numerical techniques (solution of ODEs or PDEs), depending on the preference of the candidate.

**Title:** Mathematical Combustion and Flame Instabilities

**Group:** Continuum Mechanics

**Supervisor:** Dr Joel Daou ([joel.daou@manchester.ac.uk](mailto:joel.daou@manchester.ac.uk))

**Description:** Several projects are available related to the mathematical theory of flame propagation, a fascinating multi-disciplinary area of applied mathematics involving ordinary and partial differential equations; combustion basics will be introduced to candidate. The approach will typically adopt a combination of analytical techniques (asymptotic methods) and/or numerical techniques (solution of ODEs or PDEs). The multi-disciplinary experience in combustion involved will be useful for tackling research problems in other fields of application, and will constitute a valuable asset for jobs in industry (such as the automobile or the aeronautics industry). Depending on the preference of the candidate, each of the projects can be tailored in its scope and the methodology of study.

**Suggested sample projects:**

**Ignition in a flow field** (such as a Poiseuille flow) and in mixing-layers. The main aim is to determine the critical energy of the initial hot kernel (or spark) to ignite a flowing reactive mixture.

**Propagating Flames and their Stability:** This involves the investigation of the various instabilities of flames using analytical and/or numerical approaches. The flames will be modelled as travelling wave solutions to reaction-diffusion-convection equations, which may, or may not, include full coupling with the hydrodynamics (the Navier-Stokes equation).

**Flame initiation and propagation in spatially non-uniform mixtures:** This is a problem of considerable interest in combustion, whenever the reactants are spatially separated. The approach will be based on asymptotic and/or numerical methods. The Combustion basics needed for the projects will be provided and explained to the candidate.

**Title:** Mathematical analysis of acoustic wave scattering in finite metamaterials.

**Group:**

**Supervisor:** Dr Anastasia Kisil ([anastasia.kisil@manchester.ac.uk](mailto:anastasia.kisil@manchester.ac.uk))

**Description:** The project concerns analytical methods of wave scattering problems in metamaterials with edges. Research work on the project will involve new mathematical analysis of the wave propagation problems and implementation of the developed analytical methods.

Metamaterials are usually defined as engineered materials which exhibit properties not usually found in nature such as negative refractive index, acoustic filters and even cloaking. In other words they can control, direct, and manipulate sound waves in ways that were not possible before. Metamaterials are usually modelled through the periodic arrangement of some unit cells in a 3-D (metamaterials) or a 2-D fashion (metasurfaces). Many different unit cells can be created and they can be arranged in different manner in order to suit the particular application. It is a major challenge to pick the optimum configuration. Analytic mathematical methods are particularly suited to this challenge being an inexpensive way of rapidly exploring different possibilities of design. They also offer insights into the underlining physical mechanism and hence the key to tailored adaptations.

The advertised post is for a PhD student to work with Dr Anastasia Kisil within the Waves in Complex Continua Group in the Department of Mathematics at Manchester. The appointee will develop new a theoretical framework for understanding the effects of edges in acoustic metamaterials. There has been extensive research into how waves propagate within a metamaterial (periodic materials) but there has been substantially less theoretical results about the way waves interact with the edges of the metamaterials. These results are important, have many applications in noise reduction and naturally build upon the mathematical methods used for canonical scattering problems. There is also the possibility to explore the aeroacoustic uses of these metamaterials.

This post is fully funded via Kisil's Royal Society Dorothy Hodgkin Research Fellowship and the Department of Mathematics at Manchester.

**Entry requirements:** We are looking for an enthusiastic and highly-motivated graduate with

- obtained or working towards a 1st class degree in Mathematics or a closely related discipline with strong mathematical component (Master's level or equivalent);
- a solid background in complex methods, partial differential equations and wave scattering;
- good programming skills;
- good communication skills (oral and written).

Informal email queries should be directed to Dr Kisil at [anastasia.kisil@manchester.ac.uk](mailto:anastasia.kisil@manchester.ac.uk) in the first instance. Formal applications can then be submitted online.

<https://www.manchester.ac.uk/study/postgraduate-research/programmes/list/05305/phd-applied-mathematics/>

As well as transcripts and references, applicants should supply a cover letter describing their academic background and motivation for the project, as well as a complete CV (two pages maximum).

**Title:** Multi-Scale Modelling of Composite Materials

**Subject areas:** applied mathematics, material science, continuum mechanics, molecular dynamics.

**Main supervisor:** Professor William Parnell ([William.J.Parnell@manchester.ac.uk](mailto:William.J.Parnell@manchester.ac.uk))

**Co-supervisor:** Dr. Paola Carbone

**Description:** Accurate modelling of the mechanical behaviour of elastomeric materials including microscale filler particles represents a challenge in terms of the range of length scales involved. This difficulty arises because the macroscopic continuum-scale properties are strongly influenced by chemical and physical interactions at the atom-scale and mesoscale. Modelling of these materials is affected by the choice of filler particles, the particle size distribution, the percentage filler content in addition to manufacturing processes employed for the bulk material.

This PhD project will tackle example material systems using a combination of molecular dynamics computation and continuum modelling. The aim is to characterise the local mechanical properties that result from interaction between the microscale filler particle surfaces and the surrounding polymer. An example elastomer of interest is PDMS, whereas the filler particles may be glass, a thermoplastic, or an inorganic species. These properties will inform a description of the interface regions around filler particles, the properties of which vary according to distance from the filler surfaces and the nature of the surrounding polymer. The localised mechanical parameters and the nature of the interface region will then be used to develop constitutive relations within micro-mechanical models of the composite and homogenization from the mesoscale to the continuum scale, allowing the prediction of the linear elastic (low strain deformation) region of stress / strain curves. These models could potentially aid in the design and down-selection of new materials with optimised or novel properties.

The project will involve a combination of computational and modelling work. A suitable candidate should have (or be expected to soon obtain) a good degree in a STEM subject, and be willing to engage with existing computational physics / chemistry codes and techniques to tackle the molecular dynamics aspects. The scale-up process and micromechanical modelling will also require a student with strong analytical skills.

This PhD is partially industry funded.

**Start date:** Sept. 2020.

**Funding:** Funded Project (European/UK Students Only)

**Deadline:** open

**Title:** Control of bubble dynamics in Hele-Shaw Cells

**Group:** Continuum Mechanics

**Supervisors:** Dr Alice Thompson ([alice.thompson@manchester.ac.uk](mailto:alice.thompson@manchester.ac.uk)) and Prof Anne Juel

**Description:** The shape of a propagating confined bubble is a nonlinear dynamical system, with evidence of interesting transient behaviour. For low flow rates, a single centred solution is observed experimentally, but at higher flow rates the observed behaviour is more complex, and the dynamics can be very sensitive to noise. A classical simplified model for this system predicts that multiple solutions should exist, with one unstable and the others not. While we do have evidence that in certain regimes the model is a good qualitative predictor of system behaviour, this does not necessarily imply that the predicted solution multiplicity and bifurcation structure will also hold in experiments.

The aim of this project is to use methods based on feedback control to gain new insights into the system. Firstly, we will use control-based continuation (CBC) to detect and stabilise unstable solutions in the system. CBC offers a model-free method to explore stable and unstable dynamics of nonlinear dynamical systems. This project would be one of the first to apply CBC in soft matter systems and offers a novel perspective to traditional modelling and observation. Secondly, we will consider how controls can trigger new instabilities to explore highly transient behaviour.

This project would be best suited to a student with previous experimental experience or a strong interest in the interplay between experiments and modelling. Experiments would be hosted in the Manchester Centre for Nonlinear Dynamics (MCND) laboratory, and the project would be co-supervised by Prof Anne Juel.

**Title:** Numerical Computation of Fox H Functions

**Group:** Numerical Analysis and Scientific Computing

**Supervisors:** Dr Yanghong Huang ([yanghong.huang@manchester.ac.uk](mailto:yanghong.huang@manchester.ac.uk)) and Dr Marcus Webb ([marcus.webb@manchester.ac.uk](mailto:marcus.webb@manchester.ac.uk))

Most elementary or special functions can be expressed in terms of generalised hypergeometric functions, and can be evaluated by various efficient algorithms, most of which are due to the simple recursive relations between their series expansion coefficients and integral representations. During the past few decades, new special functions arise in the study of anomalous transport and fractional differential equations. These less known functions are called Fox H functions, including the Mittag-Leffler functions as the simplest example generalising the exponential function. Most techniques used in classical algorithms become less efficient, and sometime are even infeasible. In this project, insights from other disciplines like complex analysis and fractional differential equations will be explored for the efficient numerical computation of Fox H functions, focusing mainly on those arising from practical problems in fractional diffusion.

**References:**

Garrappa R. Numerical evaluation of two and three parameter Mittag-Leffler functions. SIAM Journal on Numerical Analysis. 2015;53(3):1350-69.

Mathai AM, Saxena RK, Haubold HJ. The H-function: theory and applications. Springer, 2009.

**Title:** Modelling of graphene-based actuating membranes for nano-electro-mechanical devices

**Supervisor:** Professor Matthias Heil ([matthias.heil@manchester.ac.uk](mailto:matthias.heil@manchester.ac.uk))

**Co-Supervisor:** Dr. Aravind Vijayaraghavan

Graphene has a unique combination of properties that makes it an ideal membrane material for nano-electro-mechanical devices (NEMS) — low mass, high stiffness and high electrical conductivity. Recently, Dr.

Vijayaraghavan's group have demonstrated graphene-based capacitive NEMS pressure sensors that significantly outperform the state of the art. Ongoing experimental research aims to develop such NEMS devices into higher pressure and frequency regimes to cover a wide range of potential applications.

The graphene-based membranes comprise of a complex laminated structure of one or more graphene layers and one or more dielectric layers, with varying mechanical properties. A theoretical model for the deflection of these membranes under various loading conditions, and the resultant capacitive device response, will give predictive capability to accelerate the design and fabrication of optimised devices.

The aim of the project is to use mathematical modelling to provide insight into the relevant physical mechanisms and to provide guidance for the experimental design. This will involve continuum based approaches to model (relatively thick) polymer-backed membranes in which the graphene layer contributes little to the mechanical stiffness, as well as multiscale approaches which will be employed to derive macroscopic representations of the behaviour of pure graphene membranes. The approach will be primarily theoretical/computational but a close interaction with experimental work in the NGI is anticipated.

**Title:** Mathematical Physiology and Transport in Complex Media

**Groups:** Continuum Mechanics; Mathematics in the Life Sciences; Uncertainty Quantification and Data Science

### Supervisors

- Dr Igor Chernyavsky ([Igor.Chernyavsky@manchester.ac.uk](mailto:Igor.Chernyavsky@manchester.ac.uk))
- Professor Oliver Jensen ([Oliver.Jensen@manchester.ac.uk](mailto:Oliver.Jensen@manchester.ac.uk))

**Description:** Human organs and tissues are typically very complex multi-componental systems, where geometry and physics are subtly interlinked at different spatial scales. Despite recent advances in experimental techniques and three-dimensional imaging, there is still poor understanding of structural determinants of function in health and disease. Two examples of significant clinical interest are the human placenta, a fetal life-support system, and the human lung, which are implicated in numerous pathologies that cost billions of pounds to the healthcare sector. Both systems are characterised by irregular networks embedded in a porous tissue that manage exchange of a wide spectrum of gases, nutrients and wastes. Computational approaches alone are incapable to capture the immense complexity of the tissues, and new mathematical tools are needed to systematically characterise transport in these media.

The project will suit a student who has enthusiasm to apply their skills to a challenging biomedical application with the potential to transform clinical practice.

### References

1. Jensen OE & Chernyavsky IL (2019) Ann Rev Fluid Mech 51:25 ([doi.org/10.1146/annurev-fluid-010518-040219](https://doi.org/10.1146/annurev-fluid-010518-040219)).
2. Erlich A, et al (2019) Sci Adv 5:eaav6326 ([doi.org/10.1126/sciadv.aav6326](https://doi.org/10.1126/sciadv.aav6326)).

**Funding:** Open to all. Funding is available and would provide fees and maintenance at RCUK level for home/EU students. Competitive bursaries are also available for overseas students to fully cover both fees and maintenance at RCUK level.

**Title:** Frame-based Discretisation of Differential Equations

**Group:** Numerical Analysis and Scientific Computing

**Supervisor:** Dr Marcus Webb ([marcus.webb@manchester.ac.uk](mailto:marcus.webb@manchester.ac.uk))

**Description:** One successful approach to the numerical solution of differential equations which model many real-world phenomena, is Spectral Methods. Spectral Methods involve approximating the solution using a basis of increasingly oscillatory fundamental modes. While successful when it can be applied, the applications in which spectral methods are possible are limited; in particular, when the domain of the differential equation has corners or complicated structure.

In order to design spectral methods for new applications, this project will explore removing a key assumption --- we will not use a basis of oscillatory modes, but a frame. A frame is like a basis but does not have linear independence, so there is an inherent redundancy in the solution space. This redundancy gives us much more flexibility in designing our method, but it also leads to challenges in the computation of the solution.

The project will combine ideas from Numerical Linear Algebra for the design of the novel methods and Approximation Theory for the analysis of the numerical solution.

Programming skills in MATLAB, Python, Julia or another similar language is essential.

## Pure Mathematics and Logic

Below are the projects that the Department of Mathematics offers for PhD study within the areas of Pure Mathematics and Logic.

For further information, please feel free to contact [pgr-maths@manchester.ac.uk](mailto:pgr-maths@manchester.ac.uk).

**Title:** Thermodynamic Quantum Chaos and large networks

**Group:** Analysis and Dynamical Systems

**Supervisor:** Dr Tuomas Sahlsten ([tuomas.sahlsten@manchester.ac.uk](mailto:tuomas.sahlsten@manchester.ac.uk))

**Description:** Quantum chaos is a field that is aimed to study the properties of eigenfunctions of the Laplacian (stationary quantum states) on a Riemannian manifold using the chaotic properties of the underlying geodesic flow in high energy, that is, in the large eigenvalue limit. In this sense the field is connecting quantum mechanics to classical mechanics. A key result in the field is the Quantum Ergodicity Theorem of Shnirelman, Zelditch and Colin de Verdière, which is an equidistribution result of the eigenfunctions for large eigenvalues when the geodesic flow is ergodic. In our recent work we have been attempting to study the theory for a problem of Thermodynamic Quantum Ergodicity (TQE), where instead of large energy, we fix an energy window and vary the geometric properties of the manifold such as volume or genus. The project would aim to develop TQE, in particular for the context of Lie groups and variable curvature manifolds. Moreover, we would attempt to find connect the ideas from TQE to discrete analogues such as spectral theory of large networks, which are well-developed by the recent works of Anantharaman, Brooks, Le Masson, Lindenstrauss, Sabri, and others.

**Title:** Additive combinatorics in dynamics and spectral theory

**Group:** Analysis and Dynamical Systems

**Supervisor:** Dr Tuomas Sahlsten ([tuomas.sahlsten@manchester.ac.uk](mailto:tuomas.sahlsten@manchester.ac.uk))

**Description:** The high-frequency asymptotics of Fourier coefficients of functions and measures describe their local structure. For example, they can be used to yield geometric or arithmetic features of the object under study such as on dimension, curvature, equidistribution or combinatoric structure. In my recent work I have been working on finding conditions based on ergodic theory, dynamical systems and

stochastics which yield efficient estimates for Fourier transforms of dynamically or randomly constructed objects. Furthermore, we aim to use these estimates to obtain new arithmetic/geometric applications for them. Recent revolutions on applications of additive combinatorics to dynamics by Bourgain, Dyatlov, Hochman, Shmerkin et al. have presented many interesting problems that we will attempt to now solve in the context of thermodynamical formalism. Specific projects include establishing connections between nonlinearity and Fourier transforms, and Fractal Uncertainty Principle for Gibbs measures of Kleinian group actions, which would yield to new essential spectral gap estimates for Laplacian on hyperbolic manifolds.

**Title:** Scenery flow and fine structure of fractals

**Group:** Analysis and Dynamical Systems

**Supervisor:** Dr Tuomas Sahlsten ([tuomas.sahlsten@manchester.ac.uk](mailto:tuomas.sahlsten@manchester.ac.uk))

**Description:** After the recent influential works of Furstenberg and Hochman-Shmerkin techniques based on magnifying measures (and taking their tangent measures) have been essential in the study of arithmetic and geometric features of sets and measures in new settings. For example, these approaches work well with notions that involve questions on the entropy or dimensions of a measure, projections and distance sets, or features related to equidistribution. The key ideas are based on the dynamics or stochastics of the process of magnification and applying classical tools from ergodic theory and Markov chains. I have been recently working on developing these techniques with new arithmetic and geometric applications in mind. The project would try to attempt develop these in the setting of nonconformal dynamics, in particular self-affine fractals such as Baranski carpets and also fractals arising from nonsmooth dynamics such as quasiregular geometry.

**Title:** Open Maps

**Group:** Analysis and Dynamical Systems

**Supervisor:** Dr Nikita Sidorov ([nikita.a.sidorov@manchester.ac.uk](mailto:nikita.a.sidorov@manchester.ac.uk))

**Description:** The study of open maps is an exciting relatively new area of the theory of dynamical systems. The characterisation of the holes involves geometry ('shape') and measure theory ('size'), so their study involves a complex interplay between dynamics, geometry and analysis.

The standard dynamical approach is to assume that the survivor set (= the points whose orbits do not fall into a hole) is "sufficiently large" and investigate ergodic and geometric

properties of the induced map. The novelty of the proposed project is to take a step back and give sufficient conditions that the survivor set is indeed "sufficiently large" (uncountable, say) for the induced map to be meaningful. This involves a detailed analysis of the class of holes under investigation.

The classes of maps under investigation include - but are not limited to - expanding maps of the interval, algebraic toral automorphisms and subshifts.

**Title:** Model Theory of Fields with Operators

**Group:** Mathematical Logic

**Supervisor:** Omar Leon Sanchez ([omar.sanchez@manchester.ac.uk](mailto:omar.sanchez@manchester.ac.uk))

**Description:** Model theory is a branch of Mathematical logic that has had several remarkable applications with other areas of mathematics, such as Combinatorics, Algebraic Geometry, Number Theory, Arithmetic Geometry, Complex and Real Analysis, Functional Analysis, and Algebra (to name a few). Some of these applications have come from the study of model-theoretic properties of fields equipped with a family of operators. For instance, this includes differential/difference fields. In this project, we will look at the model theory of fields equipped with a general class of operators (that unifies other known approaches) and also within certain natural classes of fields (such as real closed fields). Several foundational questions remain open around what is called "model-companion", "elimination of imaginaries", and the "trichotomy", this is a small sample of the problems that will be tackled.

**Title:** Algebraic differential equations and model theory

**Group:** Mathematical Logic/Algebra

**Supervisor:** Dr Omar Leon Sanchez ([omar.sanchez@manchester.ac.uk](mailto:omar.sanchez@manchester.ac.uk))

**Description:** Generally speaking this area is currently my main focus of research. Differential rings and algebraic differential equations have been a crucial source of examples for model theory (more specifically, geometric stability theory), and have had numerous application in number theory, algebraic geometry, and combinatorics (to name a few).

In this project we propose to establish and analyse deep structural results on the model theory of (partial) differential fields. It has been known, for quite some time now, that while the classical notions of 'dimension' differ for differential fields, there is a strong relationship between them. We aim to tackle the following foundational (still open)

question of this theory: are there infinite dimensional types that are also strongly minimal? This is somewhat related to the understanding of regular types, which interestingly are quite far from being fully classified. A weak version of Zilber's dichotomy have been established for such types, but is the full dichotomy true?

The above is also connected to the understanding of differential-algebraic groups (or definable groups in differentially closed fields). While the notions of dimension agree for these objects in the 'ordinary' case, the question is still open in the 'partial' case. We expect that once progress is made in the direction of the above problems, we will also be closer to the answer of this question.

There are classical references for all of the above concepts and standing problems, so the interested student should have no problem in learning the background material (and start making progress) in a relatively short period of time.

**Title:** Axiomatic approaches to the Hrushovski Programme

**Group:** Mathematical Logic/ Algebra

**Supervisor:** Alexandre Borovik ([alexandre.borovik@manchester.ac.uk](mailto:alexandre.borovik@manchester.ac.uk))

**Description:** The celebrated Hrushovski Programme is aimed at proving that the group of fixed points of a generic automorphism of a simple group of finite Morley rank behaves as a pseudofinite group and, with some luck, is pseudofinite indeed. The aim of the project is to analyse a few configurations where the assumptions of the Hrushovski Conjecture are strengthened. For example, an interesting case is where the fixed points sets of the automorphism in question have "size" with values in a linearly ordered ring which behaves in a strict analogy with cardinality of finite sets; will in that case the group of fixed points be pseudofinite? This question may perhaps involve some non-trivial model theory of the ring of "sizes" and some abstract versions of the Lang-Weil inequality linking the Morley rank of an invariant definable set and the "size" of the set of its fixed points.

**Title:** Morita equivalences of finite groups

**Group:** Algebra

**Supervisor:** Charles Eaton ([charles.eaton@manchester.ac.uk](mailto:charles.eaton@manchester.ac.uk))

**Description:** Most of my current research is focused on the problem of identifying Morita equivalence classes of blocks of finite groups. This is part of the study of the representation theory of finite groups with respect to fields of prime characteristic.

Briefly, Morita equivalence is an equivalence of module categories, preserving the structure of modules for an algebra.

This problem is fundamental to the area, and ties in with another of my areas of interest, global-local relationships in finite groups.

Problems range from Donovan's conjecture, which is a finiteness conjecture concerning the number of Morita equivalence classes, to classification of Morita equivalence classes in specific cases.

A tool I use frequently is the classification of finite simple groups, but there is scope for a variety of projects suited to different interests. The precise nature of the project would be open for discussion with the prospective student.

**Title:** Self-affine sets: geometry, topology and arithmetic

**Group:** Analysis and Dynamical Systems

**Supervisor:** Dr Nikita Sidorov ([sidorov@manchester.ac.uk](mailto:sidorov@manchester.ac.uk))

**Description:** Iterated function systems (IFS) are commonly used to produce fractals. While self-similar IFS are well studied, self-affine IFS are still relatively new.

In a recent paper Kevin Hare and I considered a simple family of two-dimensional self-affine sets ( $\mathbb{S}$ -attractors of self-affine IFS) and proved several results on their connectedness, interior points, convex hull and corresponding simultaneous expansions. A great deal of natural questions (simple connectedness, set of uniqueness, dimensions, etc.) remain open - even for this most natural family.

The project is aimed at closing these gaps as well as generalising our results to other 2D families (which are completely classified) as well as higher dimensions.

**Title:** Lie algebra actions on noncommutative rings

**Group:** Algebra

**Supervisor:** Yuri Bazlov ([yuri.bazlov@manchester.ac.uk](mailto:yuri.bazlov@manchester.ac.uk))

**Description:** My interests in Lie theory are focused on Lie algebra actions on noncommutative rings. An example is the representation of a Lie group, and its Lie algebra, on exterior powers of a finite-dimensional module, or on a Clifford algebra. Many methods of the Chevalley-Kostant theory still apply, but often need to be combined with tools of noncommutative algebra.

A project in this area may suit a student with background in Lie algebras and/or representation theory.

**Title:** Reflection groups in noncommutative algebra

**Group:** Algebra

**Supervisor:** Yuri Bazlov ([yuri.bazlov@manchester.ac.uk](mailto:yuri.bazlov@manchester.ac.uk))

Description: Finite linear groups generated by reflections arise in many areas of algebra, Lie theory being a prominent example. In the work of Chevalley, Shephard, Todd and Serre, reflection groups are seen to be the groups which have "good" rings of invariants when acting on a ring of polynomials.

More recently, reflection groups have been studied in connection with noncommutative rings that are obtained from commutative rings via deformation (or quantisation) construction inspired by quantum mechanics. In particular, this has led to the rich theory of Cherednik algebras.

I am interested in reflection groups, and their quantum analogues, acting on noncommutative rings arising from quantum algebra. Projects might focus on open conjectures in this area, and should be suitable for students with background, and interest, in representation theory and/or quantum groups.

## Statistics and Probability

Below are the projects that the Department of Mathematics offers for PhD study within the areas of Statistics and Probability.

For further information, please feel free to contact [pgr-maths@manchester.ac.uk](mailto:pgr-maths@manchester.ac.uk).

**Title:** StatXAI - Statistical approaches for explainable machine learning and artificial intelligence

**Group:** Statistics and its Applications

**Supervisor:** Professor Korbinian Strimmer ([korbinian.stimmer@manchester.ac.uk](mailto:korbinian.stimmer@manchester.ac.uk))

**Description:** Machine learning (ML) and artificial intelligence (AI) methodologies are now permeating many parts of science, technology, health and society. At their core these methods rely on highly complex, high-dimensional mathematical and statistical models. However, unfortunately they are generally hard to interpret and their internal decision making mechanisms are not transparent. The objective of the StatXAI project is to develop statistical tools to understand and help create explainable ML/AI methodologies and models.

Research in StatXAI will focus on four lines of work: i) to investigate advanced nonparametric and algorithmic models such as neural networks and ensemble approaches, ii) to explore diverse strategies for explainable ML/AI using statistical approaches such as LIME/Anchors [1,2], LRP [3], explainable embeddings [4] etc., iii) to develop corresponding effective algorithms and implementing them in open source software (R and Python), and iv) to deploy and test explainable models in biological and medical settings.

The University of Manchester is a partner university of the Alan Turing Institute (ATI, the UK national institute for data science and artificial intelligence). PhD students will have the opportunity to interact and engage with the ATI.

### References:

[1] Ribeiro et al. 2016. "Why Should I Trust You?": Explaining the Predictions of Any Classifier. <https://arxiv.org/abs/1602.04938> [2] Ribeiro et al. 2018. Anchors: High-Precision Model-Agnostic Explanations. <https://homes.cs.washington.edu/~marcotcr/aaai18.pdf>

[3] Montavon et al. 2018. Methods for Interpreting and Understanding Deep Neural Networks. Digital Signal Processing, 73:1-15.

<https://doi.org/10.1016/j.dsp.2017.10.011>

[4] Qi and Li. 2017. Learning Explainable Embeddings for Deep Networks.

<http://www.interpretable-ml.org/nips2017workshop/papers/04.pdf>

**Prerequisites:**

Interest in modern machine learning methods and computational statistics, knowledge of multivariate statistics, experience in programming in R and Python. Note the focus of the PhD project lies on methods and algorithms rather than on pure theory.

**Title:** Optimal Experimental Designs

**Group:** Statistics and its Applications

**Supervisor:** Alexander Donev ([a.n.donev@manchester.ac.uk](mailto:a.n.donev@manchester.ac.uk))

**Description:** The success of many experimental studies in Biology, Chemistry, Engineering, Experimental Physics, Material Science, Medicine, etc., depends on the experimental designs that are used to collect the data. The aim of this project is to develop novel statistical methods for constructing designs that have desirable statistical properties.

The applicants are expected to have deep knowledge in Statistics and Mathematics, as well as good computational skills. The focus of the project will be decided to suit the background and the strengths of the student.

**Reference:**

Atkinson, A.C., Donev, A.N. and Tobias, R.D. (2007). Optimum Experimental Designs, with SAS. Oxford University Press.

**Title:** Distributional approximation by Stein's method

**Group:** Probability and Stochastic Analysis

**Supervisor:** Dr Robert Gaunt ([robert.gaunt@manchester.ac.uk](mailto:robert.gaunt@manchester.ac.uk))

**Description:** Stein's method is a powerful (and elegant) technique for deriving bounds on the distance between two probability distributions with respect to a probability metric. Such bounds are of interest, for example, in statistical inference when samples

sizes are small; indeed, obtaining bounds on the rate of convergence of the central limit theorem was one of the most important problems in probability theory in the first half of the 20th century.

The method is based on differential or difference equations that in a sense characterise the limit distribution and coupling techniques that allow one to derive approximations whilst retaining the probabilistic intuition. There is an active area of research concerning the development of Stein's method as a probabilistic tool and its application in areas as diverse as random graph theory, statistical mechanics and queuing theory.

There is an excellent survey of Stein's method (see below) and, given a strong background in probability, the basic method can be learnt quite quickly, so it would be possible for the interested student to make progress on new problems relatively shortly into their PhD. Possible directions for research (although not limited) include: extend Stein's method to new limit distributions; generalisations of the central limit theorem; investigate 'faster than would be expected' convergence rates and establish necessary and sufficient conditions under which they occur; applications of Stein's method to problems from, for example, statistical inference.

**Literature:**

Ross, N. Fundamentals of Stein's method. *Probability Surveys* 8 (2011), pp. 210-293.

**Title:** Scheduling and Parallel Computing

**Group:** Probability and Stochastic Analysis/ Numerical Analysis and Scientific Computing

**Supervisor:** Dr Neil Walton ([Neil.Walton@manchester.ac.uk](mailto:Neil.Walton@manchester.ac.uk))

**Description:** Consider a large computational task, for instance, solving a large system of linear equations. This task can be split into many smaller jobs which are then scheduled and queued at a large number of different heterogeneous computing resources and are executed in parallel. The processing requirements of the different resources (for instance CPUs and GPUs) are different, and they may well have different communication costs. In this project we will be interested in understanding the stochastic effects before designing (and implementing) novel distributed scheduling algorithms.

We seek a student with skills in probability, optimization and mathematical modeling. A student with good programming skills (particularly in C/C++) would be preferred.

**Title:** Joint modelling of multivariate longitudinal data and time-to-event outcomes

**Group:** Statistics and its Applications

**Supervisor:** Professor Jianxin Pan ([Jianxin.pan@manchester.ac.uk](mailto:Jianxin.pan@manchester.ac.uk))

**Description:** Hypertension is widely considered to be one of the most important risk factors for cardiovascular diseases. In a typical hypertensive trial, systolic and diastolic blood pressures are measured repeatedly over time and the time to event, such as stroke, coronary heart disease and death, is recorded for each patient. When considering the effectiveness of antihypertensive drug therapy for reduction in event risk rates, most medical publications have ignored the blood pressure measurements or used only baseline values. An important question is whether the effect of antihypertensive drug therapy in reducing risk rates acts principally through blood pressure reduction.

In this project we will develop novel statistical models and methodology for joint modelling of multivariate longitudinal data and time-to-event outcomes to address this question using big data on hypertension trial from British Medical Research Council. Various practical issues including missing value problem, explanatory variable selection, measurement errors and model selection will be addressed when building the models. Computational simulations will be studied to assess the performance of the proposed methods and numerical comparisons will be made to the existing methods. Statistical modelling and mathematical development will be the focus of the project.

The University of Manchester is a partner university of the Alan Turing Institute (ATI). PhD students will have the opportunity to interact and engage with the Health and Medical Sciences team in the ATI.

References:

[1] Dai, H. and Pan, J. (2018). Joint modelling of survival and longitudinal data with informative observation times. *Scandinavian Journal of Statistics*. 45, 571-589.  
DOI: 10.1111/sjos.12314

[2] Tsiatis, A. A. and Davidian, M. (2001). A semiparametric estimator for the proportional hazards model with longitudinal covariates measured with error. *Biometrika*, 88, 447–458.

**Title:** High-dimensional statistical network and inference

**Group:** Statistics and its Applications

**Supervisor:** Professor Jianxin Pan ([Jianxin.pan@manchester.ac.uk](mailto:Jianxin.pan@manchester.ac.uk))

**Description:** In the era of big data, high-dimensional data problem occurs often in a number of fields for many reasons, for example, when the number of variables under consideration is much bigger than the sample size, i.e.,  $p \gg n$ . For high-dimensional

data, many conventional statistical inferences are no longer appropriate, for instance, the sample covariance or precision matrix (i.e., the inverse of the covariance matrix) becomes singular and conventional statistical methods fail to make reliable analysis.

In this project, we aim to propose novel statistical methods to identify the underlying covariance structure by regularizing certain covariance matrix estimators. Statistical modelling strategy will be applied to modelling of the covariance/precision matrix, such that the resulting covariance/precision matrix has a transplant structure, is positive definite and is sparse and/or fully regularised. The resulting precision matrix will be applied to statistical network studies. Various theoretical and numerical issues will be addressed. Real data sets arising in financial sectors and medical studies will be analysed using the proposed methods.

The University of Manchester is a partner university of the Alan Turing Institute (ATI). PhD students will have the opportunity to interact and engage with the ATI.

#### References:

- [1] J. Fan, Y. Liao and M. Mincheva. Large covariance estimation by thresholding principal orthogonal complements. *J Roy Stat Soc B*, **75**, (2013), 656-658.
- [2] H. Ye and J. Pan (2006). Modelling covariance structures in generalized estimating equations for longitudinal data. *Biometrika*, 93, 927-941.