Feedback on the MATH10121 exam of 2018-19

Question 1 This question tested ability to manipulate complex numbers and represent them in the complex plane as well as evaulating complex roots. It also tested sketching of functions in Cartesian and polar coordinates and integrating functions of a single variable.

(a) Almost all students were able to convince themselves that plus and minus i were roots. A handful of students contented themselves in showing that only one was a root and that the other was immediate as "all solutions of a polynomial come in conjugate pairs". Despite this, without the qualification that the coefficients need to be real, this was perhaps a little too expedient.

(b) Roughly half of students made no headway here. Some made no attempt at all, while many others made confusing attempts to write the solutions as roots of unity. The other half of students correctly realised that they could factor the polynomial, and enjoyed plain sailing from there.

(c) Accomplished well by most students.

(d) Lots of students were seemingly disoriented by the switch from Cartesian to polars. Many students tried to replace x with $r\cos\theta$ and some just repeated the same integral from Part (c). It was also disappointing to see many students forget to sketch the curve.

Question 2 This question tested the relationship between complex numbers, trigonometric and hyperbolic functions, as well as using the basic definition of the derivative and evaluating Taylor series. There was also more sketching and integration.

(a) This question was largely well answered although many students forgot to specify that one must take real parts of the expression for $\mathrm{e}^{\mathrm{i}(A+B)}$ to obtain the desired result.

(b) A large proportion of students simply wrote down the terms from the formulæ for sin and cos with no further justification that it was a Taylor series. Very few students wrote a complete and careful description of what they were doing, i.e. start with the general formula for a Taylor series; specialise to the expansion about $x = 0$; evaluate the derivatives and substitute into the general expression.

(c) Most students got the correct answers, but a large number of the sketched graphs were poor with missing labels for the functions, or the axes. Despite the correct sketch, many students did not specify that the range of $\cosh^{-1} x$ is only the positive (or negative) real numbers.

(d) A high proportion of students did not use the fundamental definition of the derivative, which should be expressed as a limit

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

Most students simply used the definition of $\cosh x$ in terms of exponentials and differentiated those. That would have been fine if the derivative of the exponential were calculated from first principles; in most cases it wasn't.

(e) Almost everybody got the correct answer, but a number of students stated the answer without any justification. The intention was to use the relationship between the derivative of a function and the derivative of its inverse. Some students confused the inverse with dividing 1 by the function: $\cosh^{-1} x \neq 1/(\cosh x) = (\cosh x)^{-1}$.

(f) A surprisingly small number of students got this correct. The integral can be converted to standard form after completing the square. Most of the problems were failing to complete the square properly. The integral can be written as

$$\int \frac{1}{\sqrt{(9x+1)^2 - 1}}\,\mathrm{d}x = \frac{1}{9}\cosh^{-1}(9x+1) + C,$$

and the result can be found from the table or the result from part (e). Many, many students forgot to add the arbitrary constant.

Question 3 The first part of this question tested the ability to apply vector methods to solving geometric problems, and the second part to manipulating trigonometric functions. Very few students scored high marks.

(a) Parts ai) and aii) were answered correctly by almost everyone and the majority of people had a good attempt at part aiii). The main issues were with parts iv), v) and vi). Probably about half of the students did not even attempt these 3 parts which were together worth 10 marks, hence the low scores. Of the students who did, most of them correctly took the cross product of AB and CD and therefore managed to find the constants b and c. Not everyone who found these values managed to show that the perpendicular distance d between the lines was equal to the given expression. Even fewer students managed to show that the distance was maximised at $\alpha = 3$, as not many thought to differentiate the given expression. I found this surprising because it could be answered even without having found the values of b and c in the previous part. Only a handful of students drew the correct graph.

(b) Most students were awarded marks for finding the partial derivatives of $g(x, y)$, setting them equal to zero and trying to find the values of x and y. The main problem here is that students were not general enough. The question asks for all critical points but a lot of students only gave a couple rather than all the integer/half integer multiples that were required. Part bii) was answered well by about half of the students. It seemed to me as though the other half did not know how to do a Taylor series expansion of a function of two variables, or just got confused under the exam pressure. I saw a lot of students writing g', g'' as if g was just a function of one variable.

Another thing I noticed is just how little explanation any of the students give when answering questions. It was often just a string of algebra with very few written words and no symbols connecting parts of the question. I found it difficult to follow peoples' arguments sometimes. I'm sure a lot of students would have gained marks if they had explained what they were trying to do, even in just a few words!

Question 4 This question tested students' abilities to perform double integrals, and overall was answered rather poorly, with an average mark of around 6/16.

(a) Many students failed to make the substitution $\mathrm{d}A = r\,\mathrm{d}r\,\mathrm{d}\theta$ and thus got the wrong answer. Other common mistakes were to set $r = 1$ in the integrand or attempt to perform the integration using Cartesian co-ordinates. A number of students couldn't identify the appropriate range of values of (r, θ) needed to describe the unit disc.

b) i) Many students either identified the upper triangular region between $x = 0$, $y = \sqrt{\pi}$ and $y = x$ (instead of the one below the diagonal) or attempted to draw the function being integrated.

ii) A large proportion of students either just switched the labelling around directly to have $\int_{x=y}^{\sqrt{\pi}} \int_{y=0}^{\sqrt{\pi}}$ (which doesn't make sense) or switched the x and y in the integral to have $\int_{x=0}^{\sqrt{\pi}} \int_{y=x}^{\sqrt{\pi}}$ (which does make sense, but is for a different region). The 1 available mark was awarded if the integral provided matched the answer to part (i), even if it was incorrect.

iii) If the order of integration was (successfully) reversed, then this integral was easy to compute and students went on to get full marks mostly. If the order of integration was not reversed then this integral was impossible to compute, with integration by parts leading to terms which cannot be integrated. Partial marks were awarded for correctly identifying antiderivatives of the functions involved.

c) i) Many students correctly identified D to be a region in the plain/area of integration and C as its boundary but very few got a second mark for stating the orientation of C correctly. A number of students stated that D and C were the same thing.

ii) A large number of students simply computed the 2D integral without using Green's Theorem. Some students mistook Q for Q_x and P for P_y and differentiated them. A number of students identified a correct pair P and Q but then got the contour integral incorrect, this was enough for half marks.

Question 5 This question tested the ability to perform partial differentiation, sometimes implicit differentiation and basic vector calculus, through div, grad and curl. It also tested the ability to find critical points of a function of 2 variables and distinguish their types, and perform some sketching of contours.

(a) A few guessed that $\left(\frac{\partial z}{\partial u}\right)_v \left(\frac{\partial u}{\partial v}\right)_z \left(\frac{\partial v}{\partial z}\right)_u = 1$ (presumably thinking the terms cancelled)— it's not equal to 1, but in fact -1. The terms don't cancel because they are each holding different variables constant.

(b) Most did this well, and a few realized that they didn't need to calculate the curl, because for any function f one has $\operatorname{curl}(\operatorname{grad} f) = 0$ (but if you calculated it and found it to be 0 you still got the mark). A few thought that div **u** is a vector — it's a scalar!

(c) (i) Several found correctly that x satisfies $x^4 + 8x = 0$ and then divided by x and lost the solution $x = 0$—that sort of error should have been drummed out of you at A-level.

(iv) Having found in (iii) that the turning point occurs at $(x, y) = (\frac{1}{2}, \frac{1}{2})$, a good number of students compared the value there with the value at other points on the line $y + x = 1$, and found correctly that the turning point is a maximum. However, many students compared that value with other points in the plane, which tells us nothing about the type of turning point *on the line.*

Overall comments: (1) a few students seem to have run out of time, but not very many.
(2) very few students wrote explanations of what they were doing (eg, "we now solve $\frac{df}{dx} = 0$"). This meant in particular partial marks for method might not have been awarded.

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