Feedback on MATH10111 Foundations of Pure Mathematics B, Feedback on January exam 2019

Rubric: Answer all eight questions in Section A. Answer four of the six questions from Section B. Almost all students chose

Question A1 [5 marks]

Learning Outcome	**Feedback**
ILO1 – Analyse the meaning of mathematical statements involving quantifiers and logical connectives, and construct the negation of a given statement.	*Similar problems in exercise sheets. ILO1 is tested at a low level in part (a), at a medium level in part (b) and at a high level in part (c).* Majority of students acheived ILO1, explaining in words the meaning of the statements given in part (a) and constructing their negations in part (b). Most were able to demonstrate their understanding further by providing a counterexample in part (c).

Question A2 [5 marks]

Learning Outcome	**Feedback**
ILO2 – Construct truth tables of simple mathematical statements and use these to determine whether two given statements are equivalent.	*Similar problems in exercise sheets. ILO2 is tested at a medium-high level.* Majority of students acheived ILO2, scoring highly on parts (a) and (c). Quite a few students struggled to remember that the converse of $p \Rightarrow q$ is simply $q \Rightarrow p$.

Question A3 [5 marks]

Learning Outcome	**Feedback**
ILO3 – Construct elementary proofs of mathematical statements using a range of fundamental proof techniques (direct argumentation, induction, contradiction, use of contrapositive).	*Similar proofs seen on exercise sheets and on midterm test. ILO3 is tested at a high level.* Majority of students acheived ILO3, constructing a proof by induction. To score full marks, this needed to be explained carefully. Common mistakes included: forgetting to argue the base case, assuming the entirety of what you wanted to prove, forgetting to introduce the notation you used, using notation inconsistently, skipping explanations. (Very similar mistakes were made on the midterm test, and detailed feedback warning against making such mistakes was provided.)

Question A4 [4 marks]

Learning Outcome	**Feedback**
ILO4 – Use basic set theoretic language and constructions, and be able to determine whether two given sets are equal.	*Similar problems in exercise sheets. ILO4 is tested at a medium level.* Majority of students acheived ILO4. The most common mistakes involved getting confused with the two types of brackets in this question.

Question A5 [4 marks]

Learning Outcome	Feedback
ILO5 – Use elementary counting arguments (pigeonhole principle, inclusion-exclusion, binomial theorem) to compute cardinalities of finite sets.	*ILO5 is tested at a low level.* Most students had not remembered the two basic definitions that were tested here. In both cases, your answers should have been phrased in teerms of bijections (see the lecture notes for the precise definitions). Most students were able to give an intuitive idea of what is meant by a set of cardinality $n \in \mathbb{N}$. However, the majority of students did not have even an intuitive grasp of what is meant by a countable set. A very common WRONG ANSWER was to say 'countable sets are finite'. Of course, this is nonsense (e.g. $\mathbb{N}$ is countable, but NOT finite.)

Question A6 [7 marks]

Learning Outcome	Feedback
ILO6 – Describe and apply basic number theoretic concepts to compute greatest common divisors and to solve linear congruences.	*Similar problems in exercise sheets. ILO6 is tested at a high level.* *ILO6 is tested at a low level.* Majority of students acheived ILO6. Students scored very highly in part (a). The most common mistakes in part (b) were: mis-remembering (and hence mis-applying) key results, and not finding all solutions.

Question A7 [5 marks]

Learning Outcome	Feedback
ILO7 – Recall formal definitions and apply these to give examples and non-examples of bijections, equivalence relations, binary operations and (abelian) groups.	*ILO7 is tested at a low level.* Majority of students acheived ILO7, and scored highly in this question. Students who did not perform well on this question made mistakes in their writing. For example, a fairly common mistake was to write 'symmetric: $aRb = bRa$', which is devoid of all meaning. Notice that the symbols a and b need to be *quantified* (e.g. $\forall a, b \in A$). Secondly, it is not clear what you intend '$aRb = bRa$' to mean; one possible way to read this would be $(aRb) \wedge (b = b) \wedge (bRa)$.

Question A8 [5 marks]

Learning Outcome	Feedback
ILO8 – Compose and invert given permutations, expressing the result in two-line notation and in cycle notation.	*Similar problems in exercise sheets. ILO8 is tested at a low level in part (b), a medium level in part (a) and at a high level in part (c).* Majority of students acheived ILO8. The most common mistakes were not providing an answer in the format requested (e.g. *disjoint* cycles, cycle notation, two line notation), and multiplying permutations in the wrong order (remember: $f \circ g$ is the function obtained by first applying g and then f).

Question B9 [15 marks]

Learning Outcome	Feedback
ILO3 – Construct elementary proofs . . . **IL07** –Recall formal definitions. . .	*Similar problems in exercise sheets. ILO3 is tested at a low level in part (b) and at a medium level in part (c). ILO7 is tested at a medium-high level.* Quite a few of you struggled with ILO7. For example, the *image* of a function $f : A \to B$ is defined to be $\{f(a) : a \in A\}$. Thus your answer to part (a) should have been a particular subset of the integers (many of you wrote down a description of the composite function instead). The majority of students achieved ILO3; most students were able to apply their knowledge of injectivity and surjectivity to correctly determine which statements in part (b) are correct, and also made a spirited attempt at part (c). The majority of mistakes in this question were due to careless writing (e.g. skipping out quantifiers and/or explanations) and confusing definitions (e.g. mixing up injective and surjective, or $\forall$ and $\exists$, composing functions in the wrong order etc.).

Question B10 [15 marks]

Learning Outcome	Feedback
ILO5 – Use elementary counting arguments . . . **ILO4** –Use basic set theoretic language and constructions. . . **ILO3** – Construct elementary proofs. . . **IL07** –Recall formal definitions. . .	*ILO3 is tested at a medium level. ILO4 is tested at a medium level. ILO5 is tested at a medium-high level. ILO7 is tested at a low level.* Majority of students acheived ILO7 and ILO3, and scored highly as a result in parts (a) and (b). Part (a) caused few problems with most students able to state the definitions, give a short proof (phrased in terms of bijections) and demonstrate understanding with a well-chosen example. A common mistake in part (b) was to state the inclusion-exclusion *formula* only; to give this formula *meaning*, you need to say what the objects are (i.e. finite sets)! In part (c) most students struggled with ILO4, making various mistakes in calculations, partcularly those involving the empty set.

Question B11 [15 marks]

Learning Outcome	Feedback
ILO7 – Recall formal definitions and apply these . . . **ILO4** – . . . be able to determine whether two given sets are equal. **ILO3** – Construct elementary proofs . . . **ILO6** – Describe and apply basic number theoretic concepts . . .	*Similar problems in exercise sheets. ILO3 is tested at a medium level. ILO4 is tested at a high level. ILO6 is tested at a low level. ILO7 is tested at a high level.* Majority of students acheived ILO7 and ILO3, and generally scored well as a result in part (a). Students struggled with ILO4; to show that sets A and B are equal one has to show that $A \subseteq B$ AND $B \subseteq A$. Part (b) tested your understanding of equivalence relations further and also your understanding of divisibility. This was mostly done well, in spite of the small typo that appeared in the question (the set $\mathbb{R}$ appeared erroneously in the definition of the equivalence relation; although it had been stated that this was a relation on the set $\mathbb{Z}$; this did not appear to cause any issues in your answers). Most mistakes made here concerned explaining why the relation is symmetric and transitive. For example, a very common answer stated '$aRb \Rightarrow 5 \mid 2a + 3b \Rightarrow 5 \mid 3b + 2a \Rightarrow bRa$' – notice that the last step has not been explained. A less common mistake involved confusing the quantifiers $\forall$ and $\exists$ when checking for reflexivity, symmetry and transitivity, with some students only showing examples in which aRb, bRc and aRc (rather than showing that *whenver* aRb and bRc we also have aRc).

Question B12 [15 marks]

Learning Outcome	**Feedback**
ILO7 – Recall formal definitions and apply these to give examples and non-examples . . . . **ILO3** – Construct elementary proofs . . . **ILO4** – Use basic set theoretic language and constructions . . . **ILO6** – Describe and apply basic number theoretic concepts . . .	*ILO3 is tested at a medium level. ILO4 is tested at a low level. ILO6 is tested at a low level. ILO7 is tested at a high level.* Majority of students acheived all ILOs in this question. The most common mistakes made were: forgetting that an identity is required to behave as such on *both sides*; mis-remembering one of the group axioms; mixing up the definitions of commutativity and associativity.

Question B13 [15 marks]

Learning Outcome	**Feedback**
ILO3 – Construct elementary proofs. . . **ILO6** – Describe and apply basic number theoretic concepts. . .	*Similar problems in examples sheets. ILO3 is tested at a medium level. ILO6 is tested at a low-medium level in parts (a) and (b) and a medium-high level in part (c).* Majority of students acheived the ILOs of this question. Most common mistakes incuded: mistakes in basic algebra when reasoning in part (a), incorrectly stating Fermat's little theorem (e.g. an incorrect formula); imprecisely stating Fermat's little theorem (e.g. a correct formula, but neglecting to explain what is p, and what is a – in particular, p has to be a prime for the statement to hold, and you need to say this); poor explanation of the basic ideas being applied in your answers to part (c).

Question B14 [15marks]

Learning Outcome	**Feedback**
ILO6 – Describe and apply basic number theoretic concepts . . . **ILO3** – Construct elementary proofs . . .	*Mostly bookwork; ILO6 is tested at a low level. ILO3 is tested at a low level in part (a), and at a high level in part (c). Part (a) has been seen on exercise sheets. A very similar proof to part (c) has been seen in lectures.* Less than half of the class attempted this question, which tested ILO6 at a low level and ILO3 at a range of levels. Most students were able to give the basic definitions in part (a)(i). Having done so part (a)(ii) should have been very easy, but many of you did not attempt this, and most students also did not remember what is meant by the fundamental theorem of arithmetic (stating the division theorem instead, or else leaving part (b) blank). The majority of students understood that a proof by contradiction was required in part (c) and made a start on this. Most people who attempted this proof were able to see the relevance of the given fact and use this to write a good argument.

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